



大连理工大学

# 基于移动可变形组件法和共形参数化技术的 复杂曲面薄壁结构拓扑优化研究

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工业装备结构分析优化与CAE软件全国重点实验室

2025-11-13

# 报告提纲

**一、研究背景与意义**

**二、相关研究进展**

**三、具体研究工作陈述**

**四、总结与展望**

# 一、研究背景与意义

# 研究背景与意义

## 工业背景



雪龙2号



和谐号



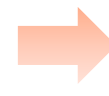
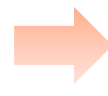
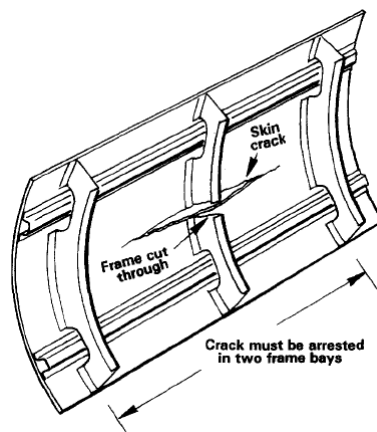
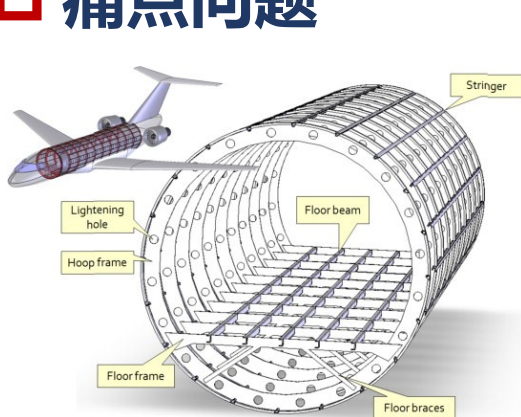
龙飞船



空客ZEROe

薄壁结构凭借其轻质高强等方面的优势，而被广泛应用于诸多重大工业装备中

## 痛点问题



对薄壁结构进行设计能够提升装备性能，但传统经验式设计已逐渐难以满足工程需求！



# 研究背景与意义

## □ 解决方案：结构优化设计

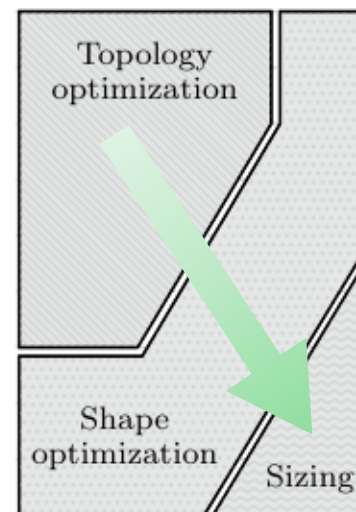
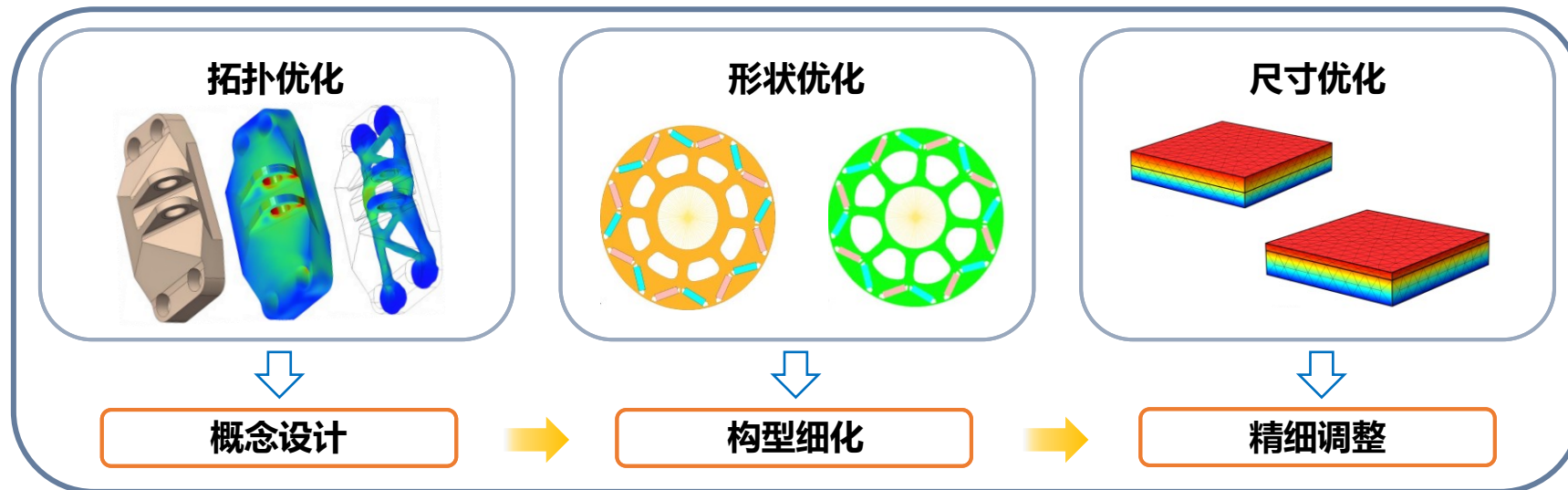


$$\begin{array}{ll} \text{minimize} & f(\mathbf{x}) \\ \text{such that} & g_j(\mathbf{x}) \geq 0, \quad j = 1, \dots, n_g, \\ & h_k(\mathbf{x}) = 0, \quad k = 1, \dots, n_e, \end{array}$$

[Haftka & Gürdal, *Elements of structural optimization*]

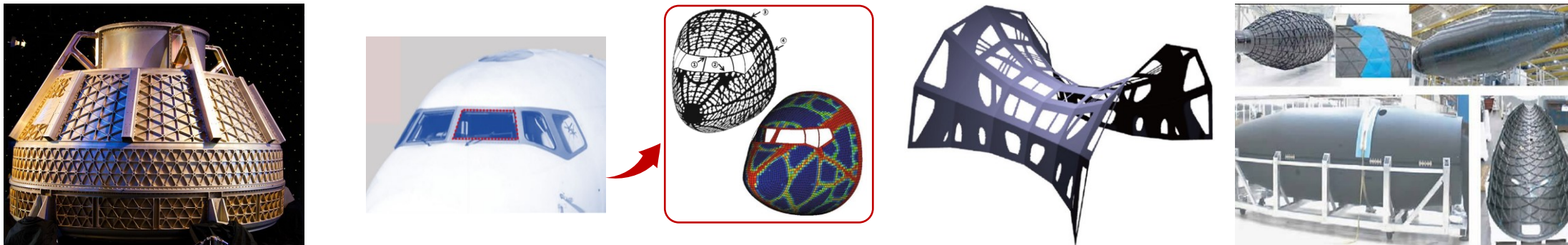
建立在严格的力学与数学理论基础，突破了传统设计中依赖工程师经验的局限性

## □ 结构优化设计的类别划分



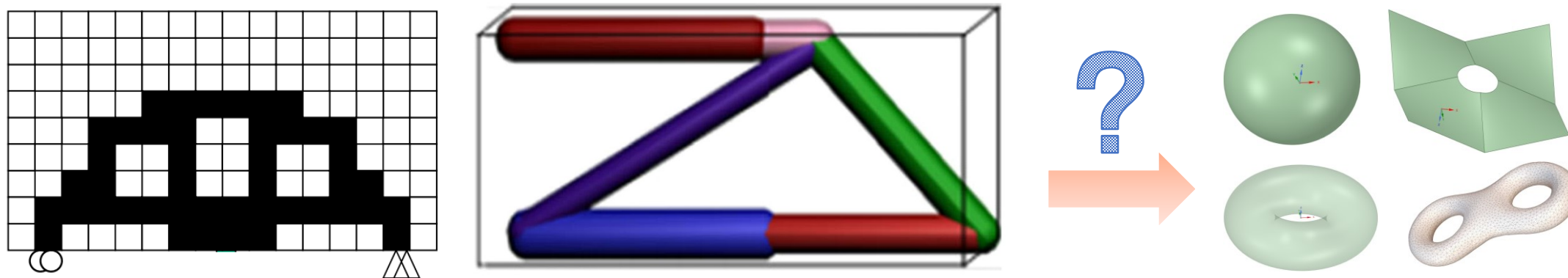
# 研究背景与意义

## □ 研究重点：薄壁结构拓扑优化



**核心：在曲面以及具有曲面特征的几何设计域上寻求材料的最优分布**

## □ 科学问题：面向复杂曲面薄壁结构的拓扑优化设计

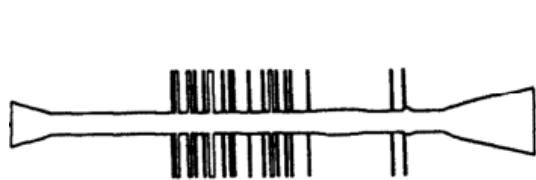


**经典方法通常基于2D/3D平直空间构建，难以应用于具有复杂曲面特征的薄壁结构**

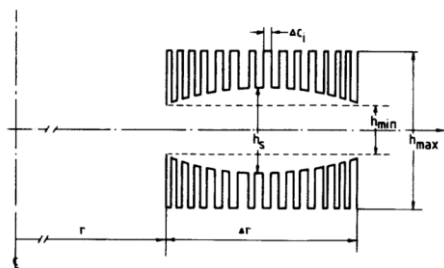
## 二、相关研究进展

# 相关研究进展 (拓扑优化方法)

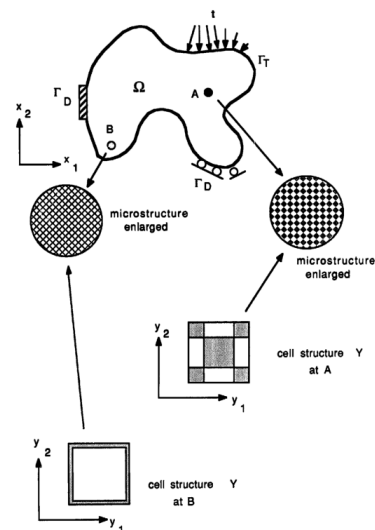
## 经典的隐式拓扑优化方法



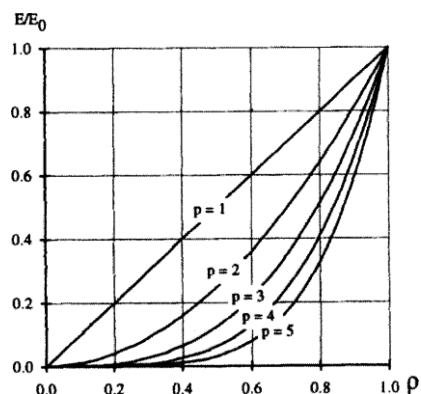
设计结果中的细小筋条



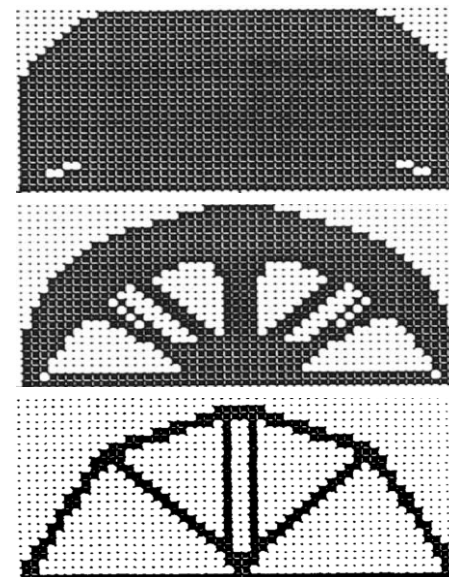
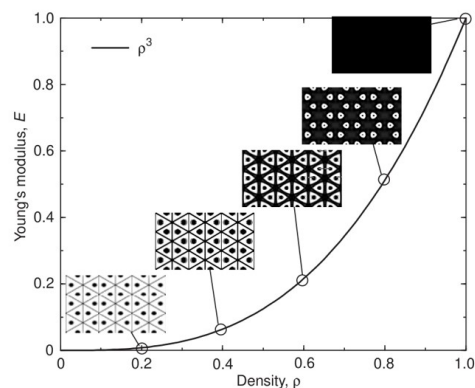
进行正则化的微结构模型



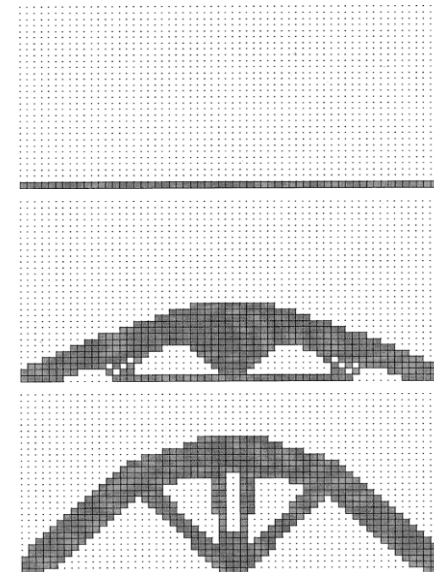
均匀化方法



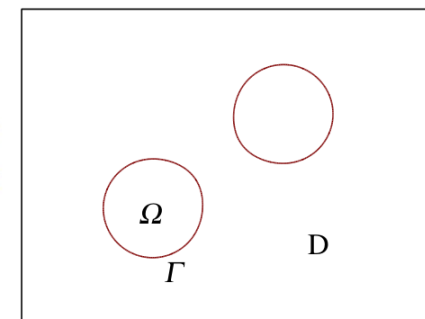
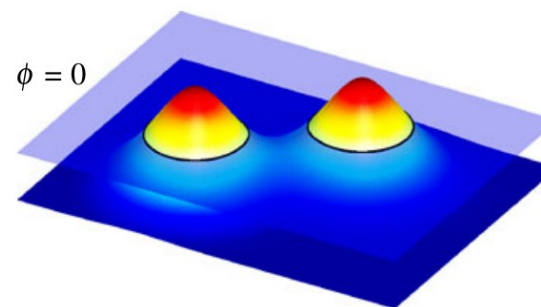
引入单元密度和材料幂律模型的SIMP法



hard-kill的ESO方法



双向的BESO方法



高维函数的等值面表示结构边界的水平集方法

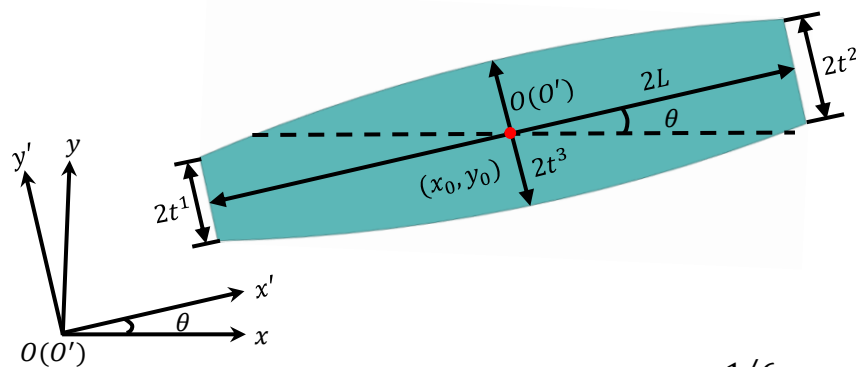


# 本文所采用的移动可变形组件法

## 基本思想 (论文2.3节)

$$\begin{cases} \phi^s(x) > 0, & \text{if } x \in \Omega_s \\ \phi^s(x) = 0, & \text{if } x \in \partial\Omega_s \\ \phi^s(x) < 0, & \text{if } x \in \Omega \setminus (\Omega_s \cup \partial\Omega_s) \end{cases}$$

拓扑描述函数(TDF)



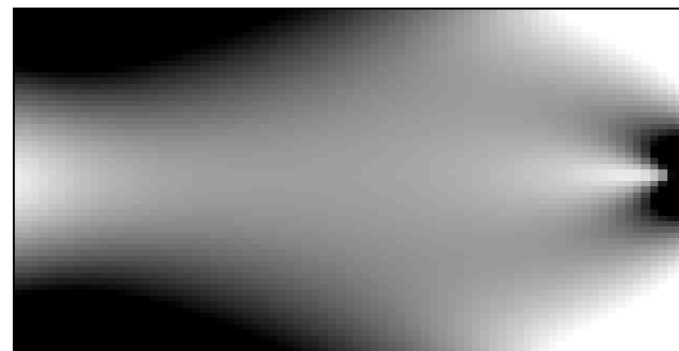
$$\phi_i = 1 - \left( \left( \frac{x'_i}{L_i} \right)^6 + \left( \frac{y'_i}{f_i(x'_i)} \right)^6 \right)^{1/6}$$

经典的超椭圆型组件描述

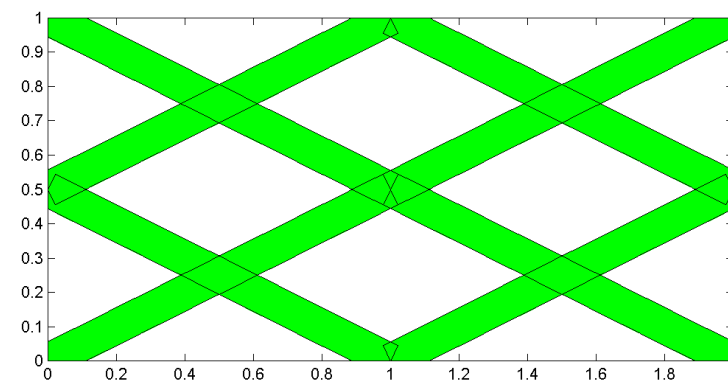
$$\phi^s = \mathcal{KS}(\phi_1, \phi_2, \dots, \phi_n) = \left( \ln \left( \sum_{i=1}^n \exp(\zeta \phi_i) \right) \right) / \zeta$$

采用KS函数进行不同组件的凝聚

## 结构优化机制



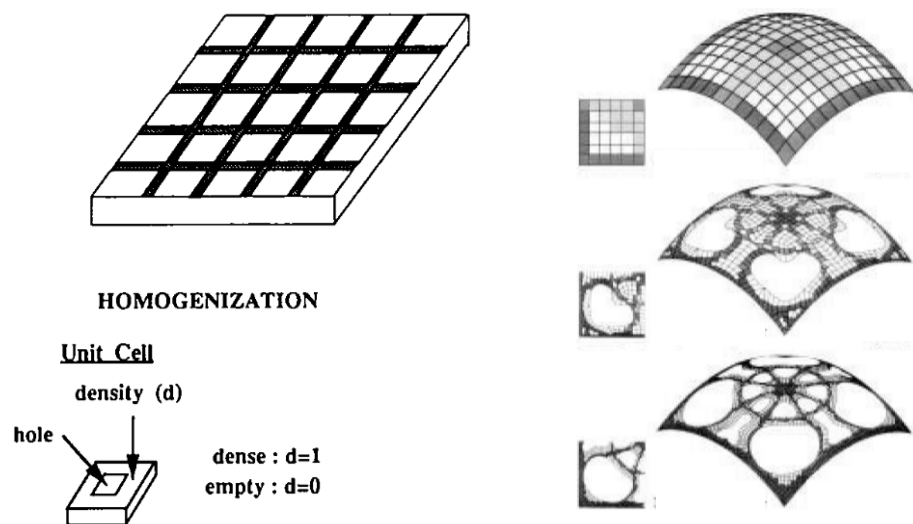
密度法：基于像素有无刻画结构拓扑



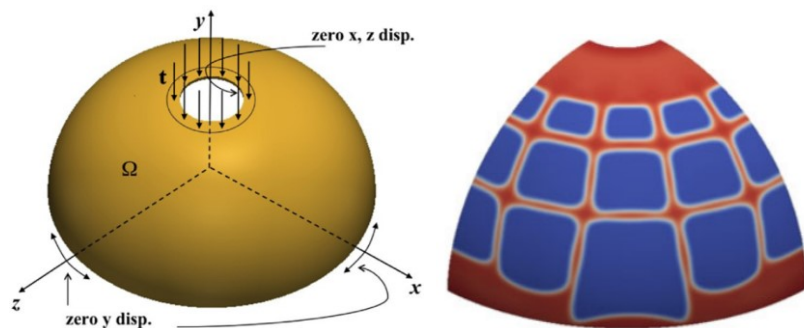
移动可变形组件法：通过组件的移动  
变形和覆盖实现拓扑变化

移动可变形组件 (Moving Morphable Component)

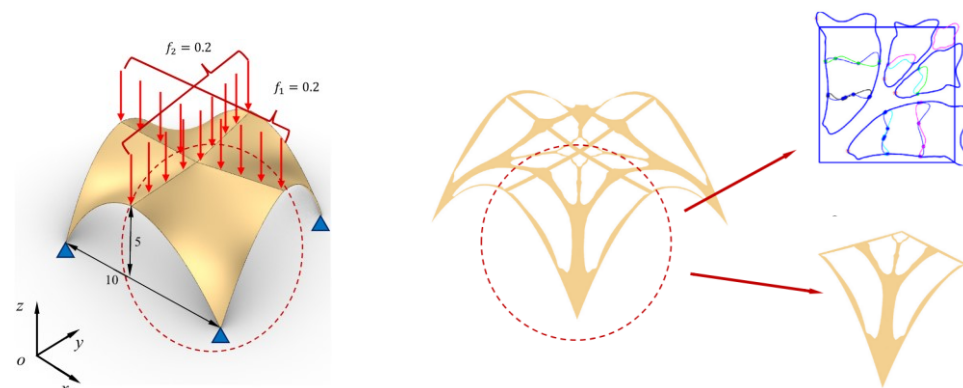
## □ 面向薄壁结构的拓扑优化研究



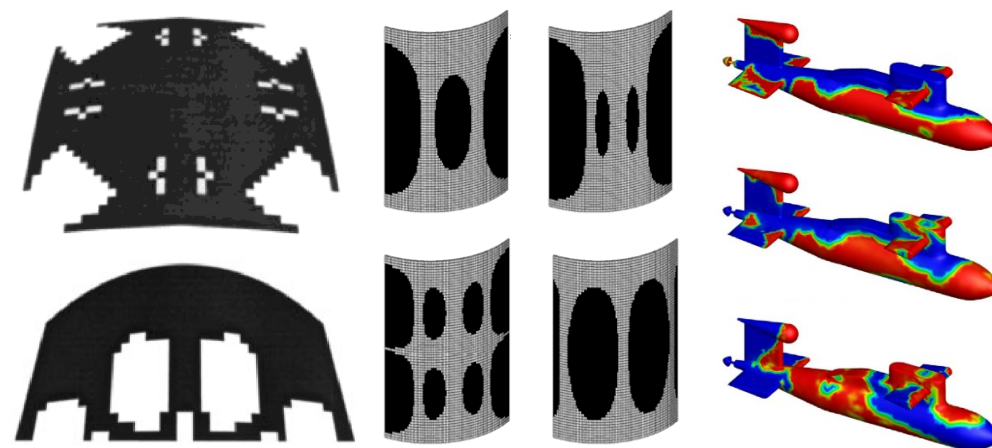
基于均匀化方法、SIMP法的薄壁结构拓扑优化



基于相场法实现薄壁结构拓扑优化



基于等几何分析方法和移动可变形孔洞法的拓扑优化



利用拓扑优化来解决一般化的薄壁结构设计问题  
(基频最大化、阻尼材料分布、表面吸声材料分布)

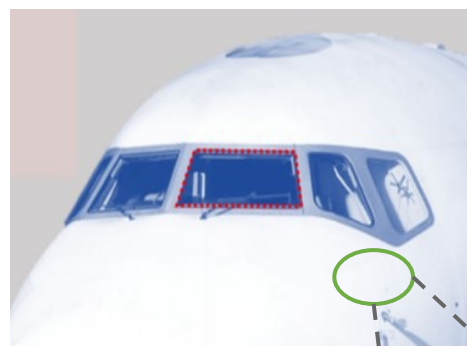
### 三、具体研究工作陈述



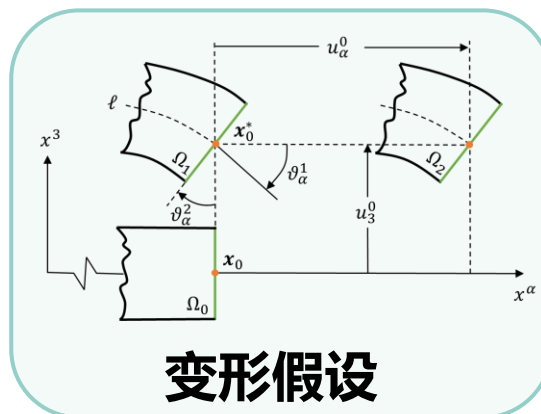
## 工作 1 曲壳拓扑优化 (第三章)

# 问题描述

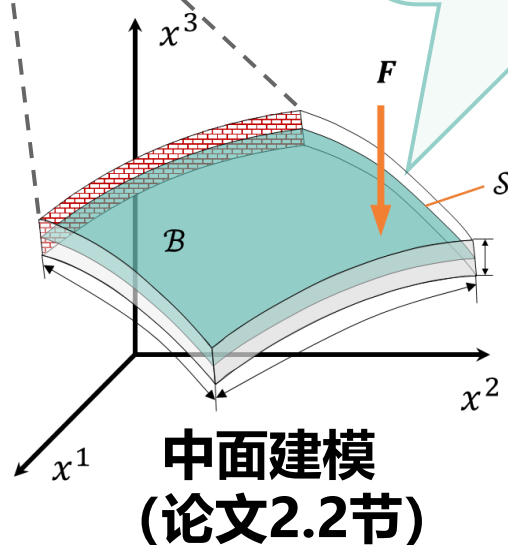
## □ 工程中通常采用壳体模型对薄壁结构进行建模



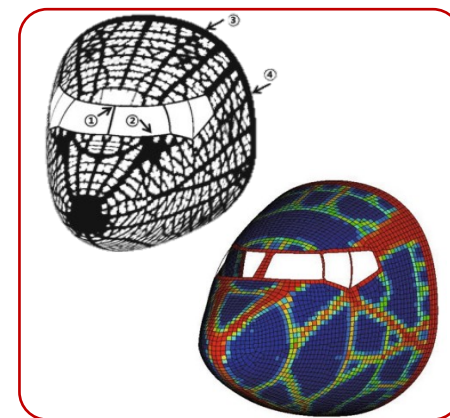
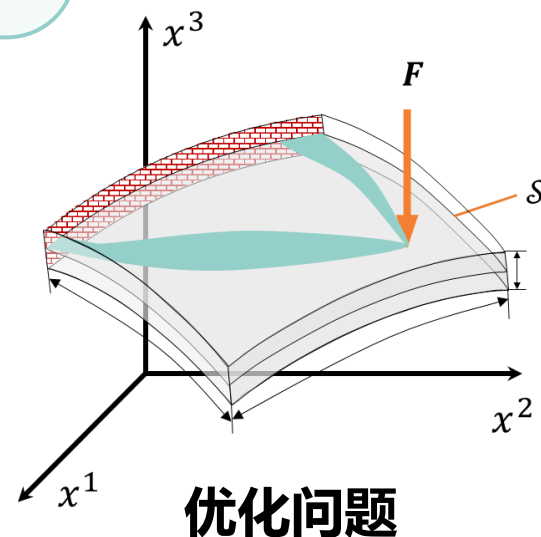
工程结构



变形假设



问题提取

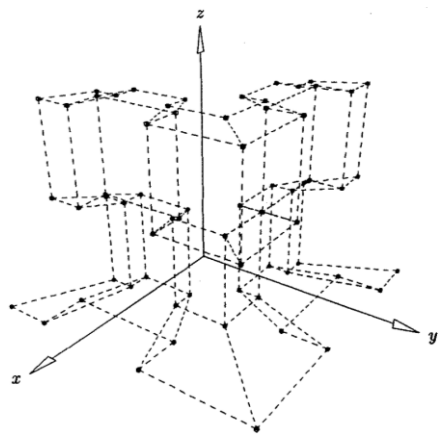


实际应用

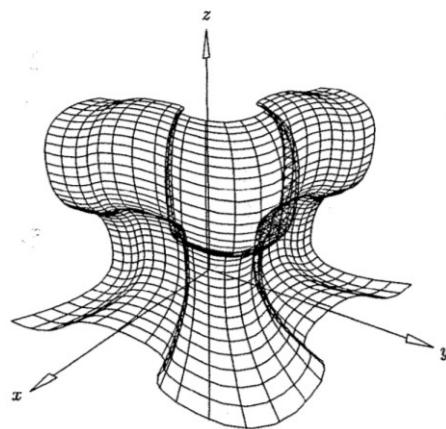
**核心问题：如何基于壳体模型进行拓扑优化，在中曲面上实现材料的最优分布？**

# 移动可变形组件法应用于壳体模型所面临的挑战

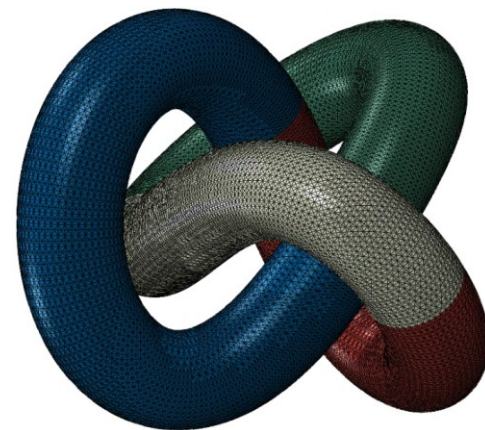
## 挑战一：如何描述复杂而又灵活的曲面



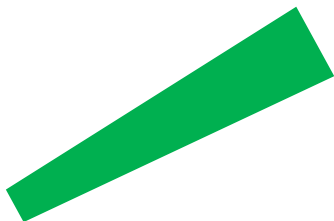
样条模型(Bezier, NURBS, T-spline...)



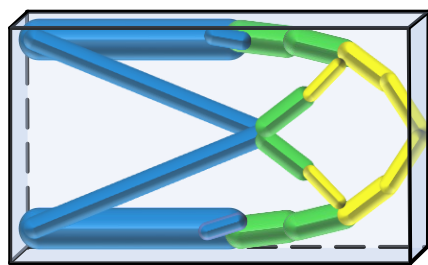
网格模型 (Triangle, Tetra, Hexahedral...)



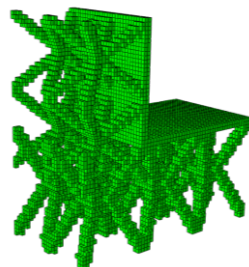
## 挑战二：如何在曲面上描述组件布局 and 材料分布



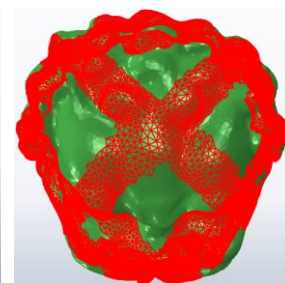
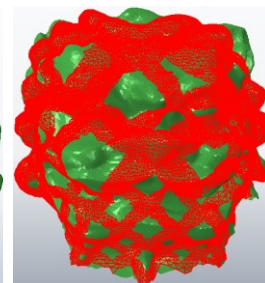
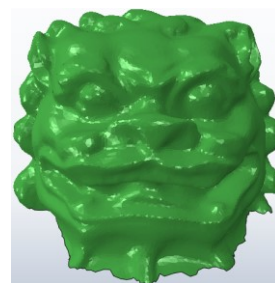
2d component



3d component



平直空间结构优化

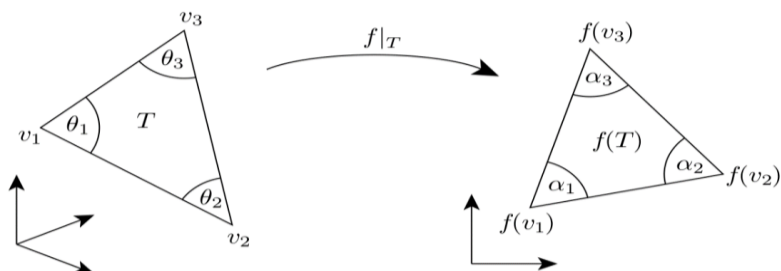
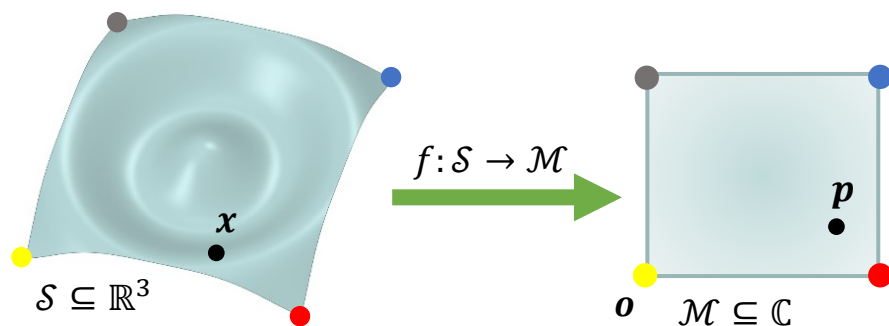


曲面上的组件分布

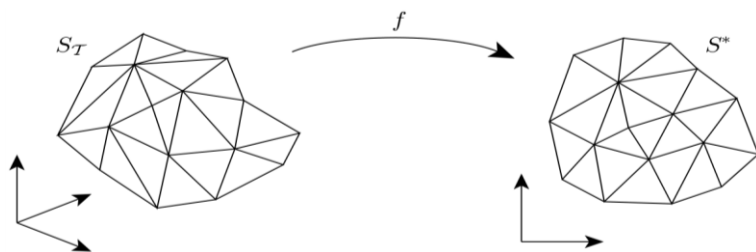
## 挑战三：如何优化曲面上分布的组件，以及如何推导解析灵敏度

# 启发：曲面参数化、共形参数化与纹理映射

## □ 曲面参数化

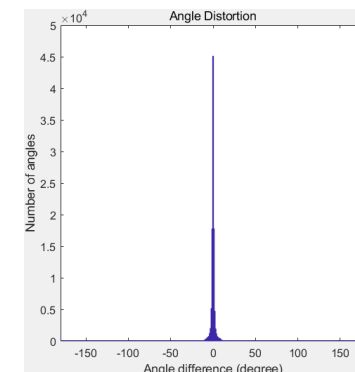
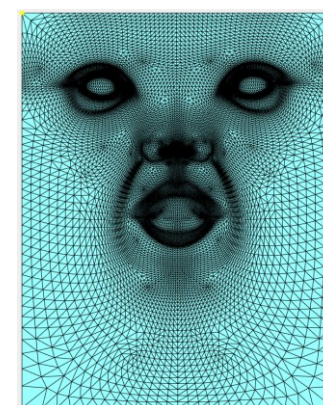
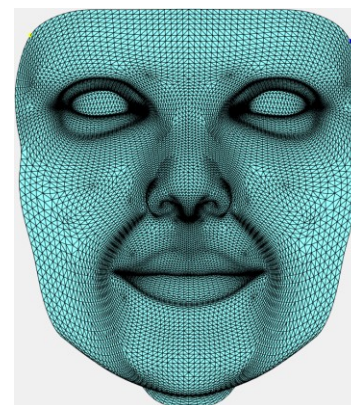


分片线性（易于求解）



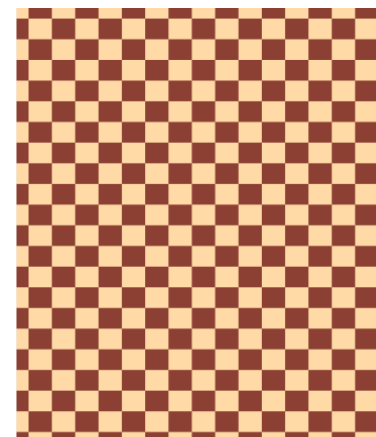
降维特性（利于应用）

## □ 共形参数化（论文2.1节）



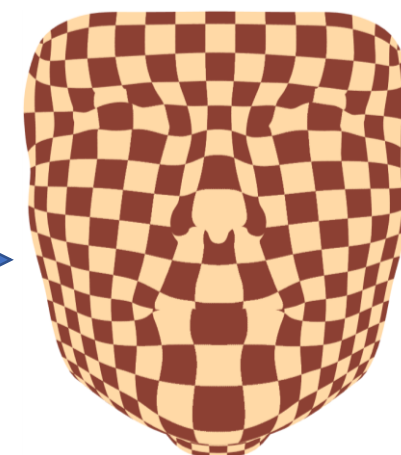
共形扭曲量的统计

共形参数化的保角特性可有效降低形状扭曲程度



纹理空间

纹理映射



物理空间

采用共形参数化技术实现的人脸模型纹理映射

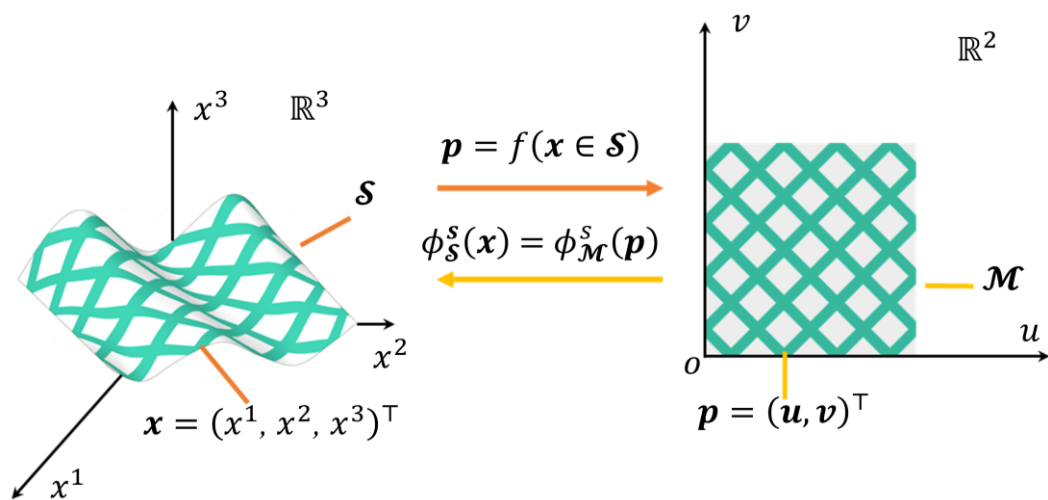
Lui et. al., *SIAM Journal on Imaging Sciences* 7.1 (2014): 391-426

Choi et. al., *SIAM Journal on Imaging Sciences* 8.1 (2015): 67-94

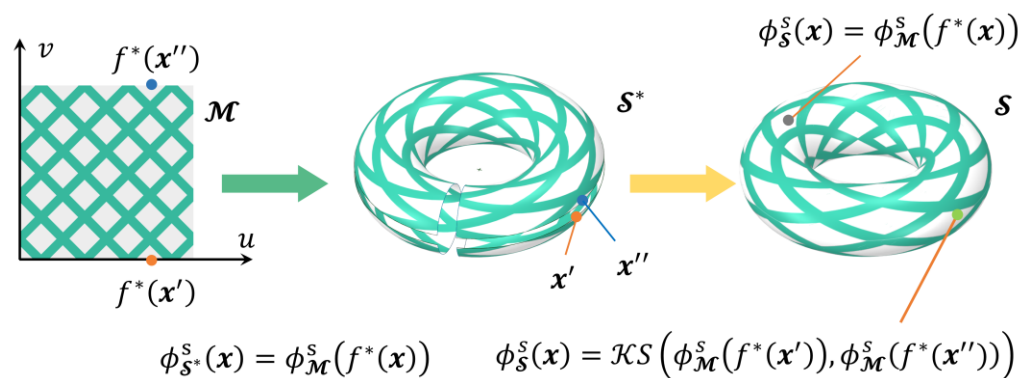


# 解决方案：曲面组件描述方法

## 简单曲面拓扑描述

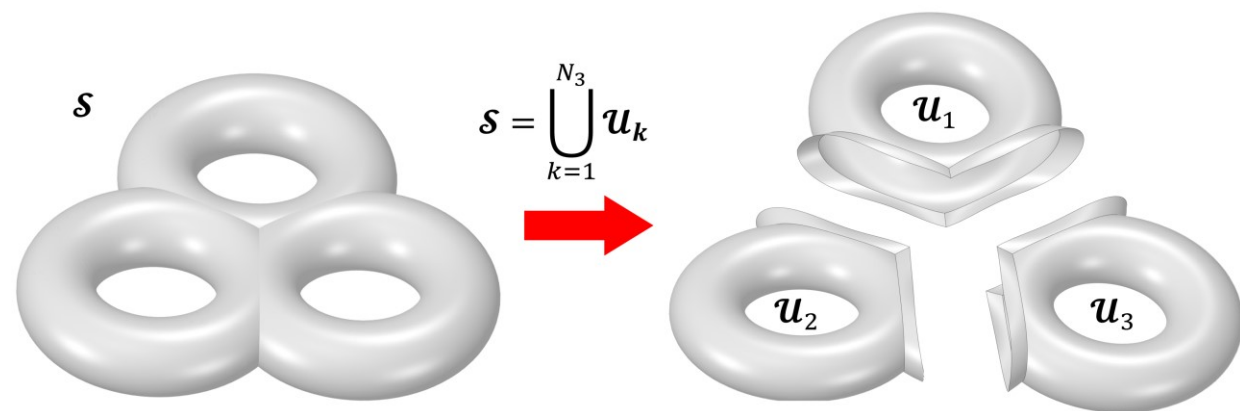


### 单连通g0开曲面的组件描述 (拓扑描述函数映射)

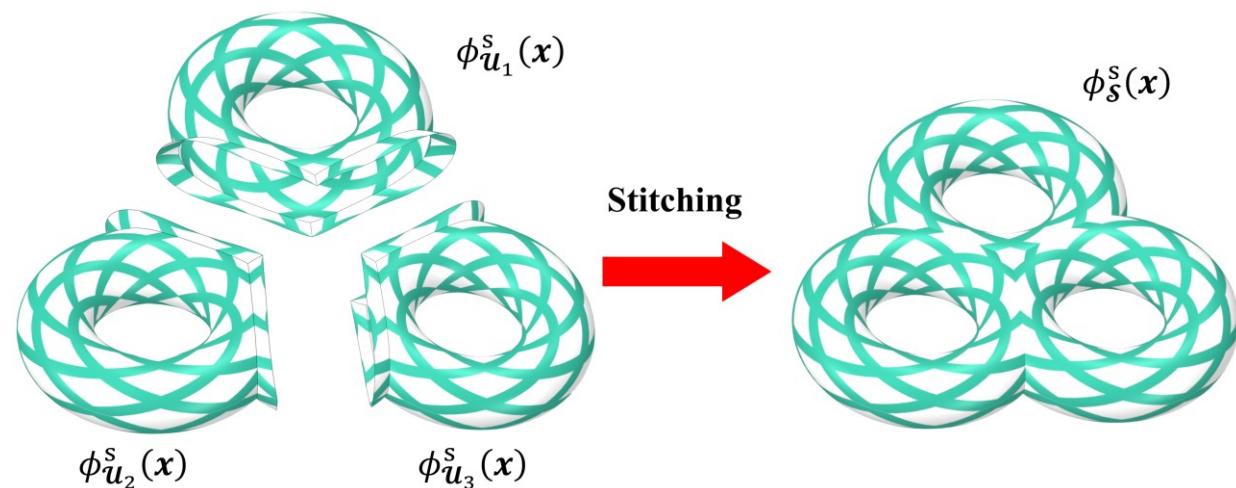


### 非零亏格曲面的组件描述 (曲面切割技术)

## 基于多面片拼接技术的复杂曲面拓扑描述



### 复杂曲面分解为有限个面片的组合

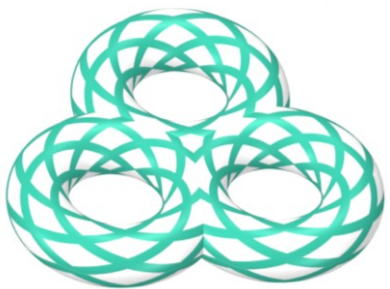


### 原始曲面拓扑描述函数由多个面片拼接得到 (Eq.3.1-3.2)



# 优化列式：设计问题的数学化表述

## 材料分布描述：从几何到力学



拓扑描述  $\phi_S^s(\mathbf{x})$

材料分布  $E(\mathbf{x}) = H(\phi_S^s(\mathbf{x}))E^s$

本构方程 
$$\begin{cases} C_0^{\alpha\beta\gamma\mu}(\mathbf{x}) = H(\phi_S^s(\mathbf{x}))C_0^{\alpha\beta\gamma\mu} \\ D_0^{\alpha\gamma}(\mathbf{x}) = H(\phi_S^s(\mathbf{x}))D_0^{\alpha\gamma} \end{cases}$$

中曲面的材料分布代表整体结构 (Eq.3.3-3.4)

## 能量的简化 (Eq.3.6-3.10)

$$\begin{cases} C = C(\mathbf{U}(\mathbf{D}), \mathbf{D}) = \int_S \int_{-\frac{t}{2}}^{\frac{t}{2}} (H(\phi_S^s(\mathbf{x}; \mathbf{D})) W_C(\mathbf{U}, \mathbf{U})) dldA \\ R_C(\mathbf{U}, \mathbf{V}) = \int_{-\frac{t}{2}}^{\frac{t}{2}} (W_C(\mathbf{U}, \mathbf{V})) dl \end{cases}$$

$$C = C(\mathbf{U}(\mathbf{D}), \mathbf{D}) = \int_S H(\phi_S^s(\mathbf{x}; \mathbf{D})) R_C(\mathbf{U}, \mathbf{U}) dA$$

沿厚度积分简化后的柔度函数

Huo, Liu\*, et. al., Guo\*, *JAM*, 2022

## 优化列式 (Eq.3.5-3.10)

Find  $\mathbf{D}, \mathbf{U}(\mathbf{x}; \mathbf{D})$

参数域中的组件几何信息

Minimize  $C = C(\mathbf{U}(\mathbf{x}; \mathbf{D}), \mathbf{D}) = \int_S \int_{-\frac{t}{2}}^{\frac{t}{2}} (\mathbf{F} \cdot \mathbf{U}(\mathbf{x}; \mathbf{D})) dldA$

S. t.

$$\left[ \int_S H(\phi_S^s(\mathbf{x}; \mathbf{D})) R_C(\mathbf{U}, \mathbf{V}) dA \right] = \int_S \int_{-\frac{t}{2}}^{\frac{t}{2}} (\mathbf{F} \cdot \mathbf{V}) dldA, \forall \mathbf{V} \in \mathcal{U}_{ad},$$

$$\left[ \int_S t H(\phi_S^s(\mathbf{x}; \mathbf{D})) dA \right] \leq \bar{V} \int_S t dA,$$

$$\mathbf{U} \in \mathcal{U}, \mathbf{D} \in \mathcal{U}_D$$

$$\mathbf{D} = (\mathbf{D}_1^\top, \mathbf{D}_2^\top, \dots, \mathbf{D}_n^\top)^\top, \mathbf{D}_i = (x_0^i, y_0^i, \theta_i, L_i, t_i^1, t_i^2, t_i^3)^\top$$

D. Chapelle, K.-J. Bathe, *The finite element analysis of shells-fundamentals*, Springer Science & Business Media, 2010.

## □ 灵敏度分析 (Eq.3.11-3.26)

$$I(\mathbf{U}, \mathbf{U}) = \int_{\mathcal{S}} H(\phi_{\mathcal{S}}^{\mathcal{S}}(\mathbf{x}; \mathbf{D})) R(\mathbf{U}, \mathbf{U}) dA$$

本研究所需要考虑的响应函数的一般形式

$$\begin{aligned} \delta I(\mathbf{U}, \mathbf{U}) &= \int_{\mathcal{S}} \delta H(\phi_{\mathcal{S}}^{\mathcal{S}}(\mathbf{x}; \delta d)) R(\mathbf{U}, \mathbf{U}) + H(\phi_{\mathcal{S}}^{\mathcal{S}}(\mathbf{x}; d)) \delta R(\mathbf{U}, \mathbf{U}) dA \end{aligned}$$

推导得一般化的响应函数关于设计变量的灵敏度表达式

$$\delta H(\phi_{\mathcal{S}}^{\mathcal{S}}(\mathbf{x}; \delta d)) = \frac{\partial H(\phi_{\mathcal{S}}^{\mathcal{S}}(\mathbf{x}; d))}{\partial \phi_{\mathcal{S}}^{\mathcal{S}}(\mathbf{x}; d)} \delta \phi_{\mathcal{S}}^{\mathcal{S}}(\mathbf{x}; \delta d)$$

此处为灵敏度分析核心部分，和二维MMC方法类似

$$\delta C = - \int_{\mathcal{S}} \delta H(\phi_{\mathcal{S}}^{\mathcal{S}}(\mathbf{x}; \delta d)) R_C(\mathbf{U}, \mathbf{U}) .$$

$$\frac{\partial C}{\partial d} = - \int_{\mathcal{S}} \frac{\partial H(\phi_{\mathcal{S}}^{\mathcal{S}}(\mathbf{x}; d))}{\partial \phi_{\mathcal{S}}^{\mathcal{S}}(\mathbf{x}; d)} \frac{\partial \phi_{\mathcal{S}}^{\mathcal{S}}(\mathbf{x}; d)}{\partial d} R_C(\mathbf{U}, \mathbf{U})$$

整理得到柔度函数关于设计变量的灵敏度

Zhang et. al., *SMO*, 53.6 (2016): 1243-1260

## □ 数值实现：从连续到离散 (Eq.3.27-3.31)

$$E(\mathbf{x}) = H(\phi_{\mathcal{S}}^{\mathcal{S}}(\mathbf{x})) E^{\mathcal{S}} \longrightarrow E_e = \rho_e E^{\mathcal{S}}$$

$$\rho_e = \sum_{i=1}^3 H_{\alpha, \epsilon}((\phi_{\Delta}^{\mathcal{S}})_{e,i}) / 3$$

$$H_{\alpha, \epsilon}(x) = \begin{cases} 1, & \text{if } x > \epsilon, \\ \frac{3(1-\alpha)}{4} \left( \frac{x}{\epsilon} - \frac{x^3}{3\epsilon^3} \right) + \frac{1+\epsilon}{2}, & \text{if } |x| \leq \epsilon, \\ \alpha, & \text{otherwise} \end{cases}$$

材料分布的具体实现 (弹性模量)  
进一步提交计算 (壳体S3单元)

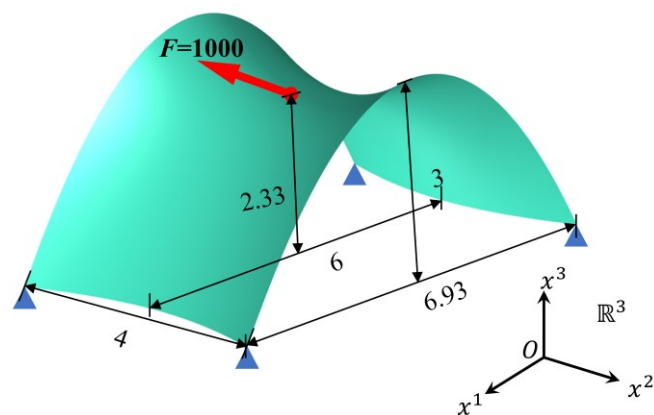


$$\left\{ \begin{aligned} C &= \sum_{e=1}^{ne} C_e = \sum_{e=1}^{ne} \sum_{i=1}^3 H_{\alpha, \epsilon}((\phi_{\Delta}^{\mathcal{S}})_{e,i}) \frac{C_e^0}{3}, \\ \frac{\partial C}{\partial d} &= \sum_{e=1}^{ne} \frac{\partial C_e}{\partial d}, \\ \frac{\partial C_e}{\partial d} &= - \frac{3C_e}{\sum_{i=1}^3 H_{\alpha, \epsilon}((\phi_{\Delta}^{\mathcal{S}})_{e,i})} \sum_{i=1}^3 \frac{\partial H_{\alpha, \epsilon}((\phi_{\Delta}^{\mathcal{S}})_{e,i})}{\partial d} \end{aligned} \right.$$

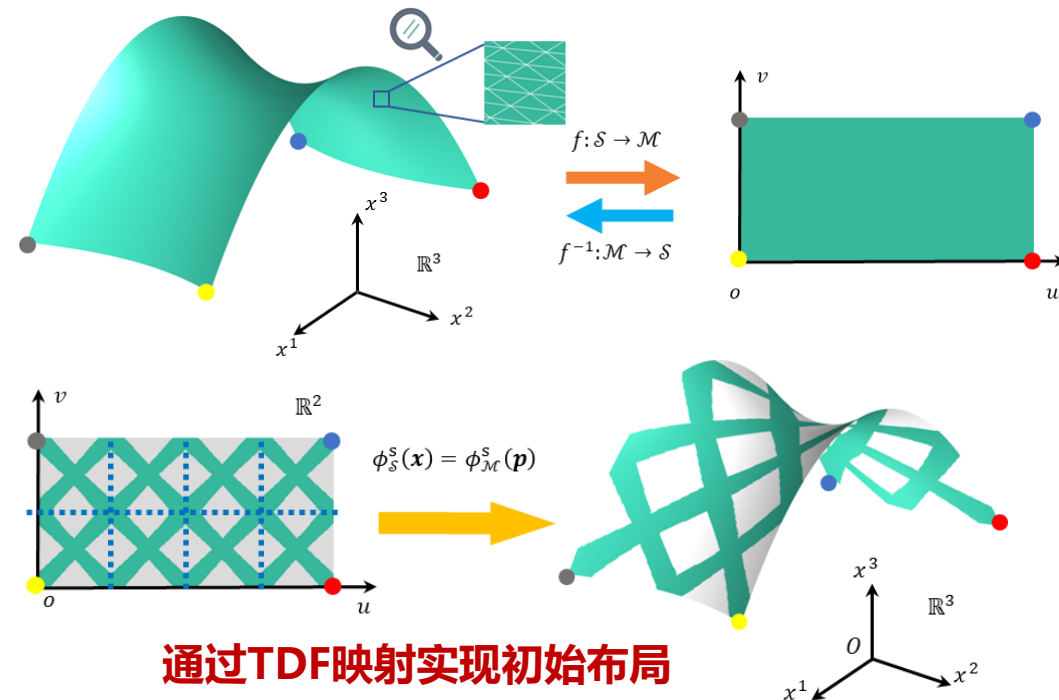
柔度灵敏度的具体计算：借助单元应变能作为媒介

# 验证算例1：马鞍面算例

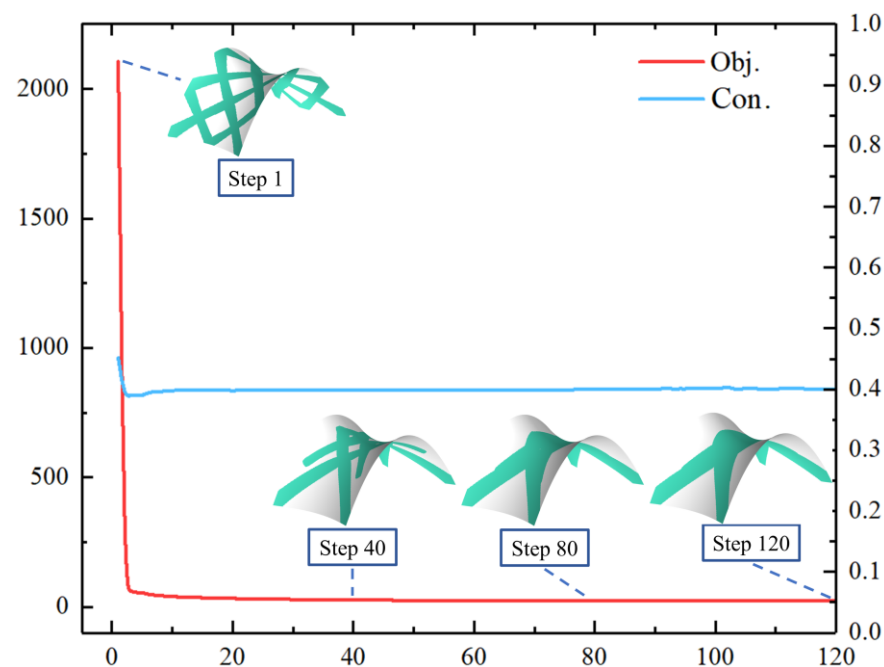
## □ 边界条件与几何尺寸



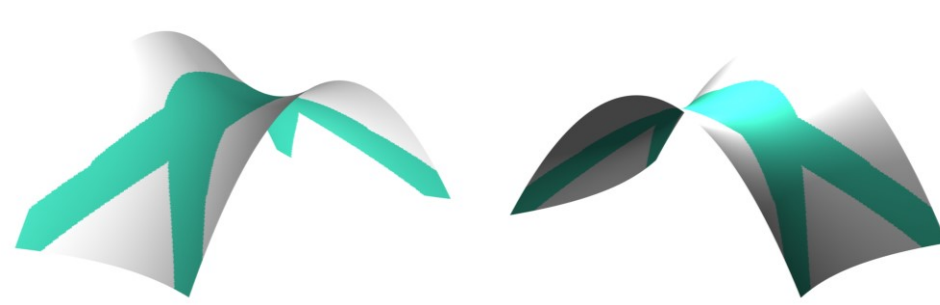
## □ 共形参数化设置与初始布局



## □ 迭代历史



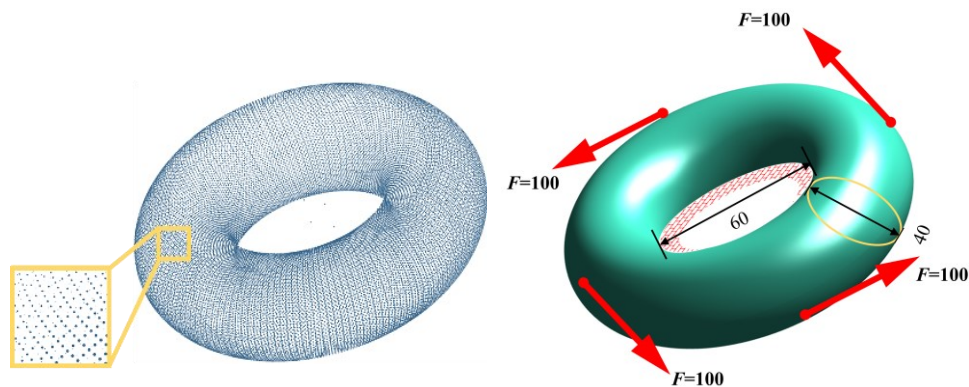
## □ 最终设计



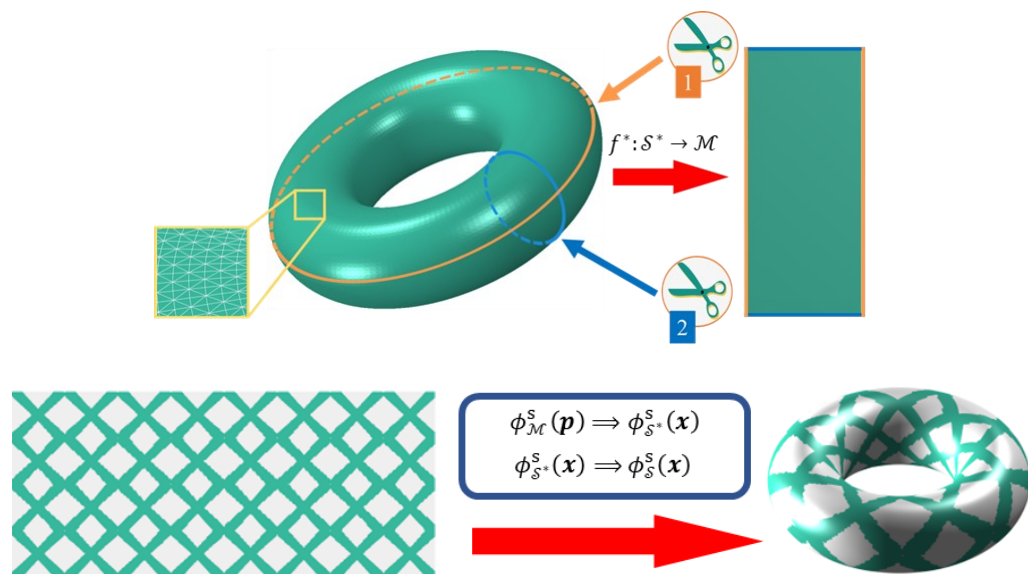
16 组件 >>> 112 设计变量

# 验证算例2：环面算例

## 点云 (.ply 文件) 与网格模型

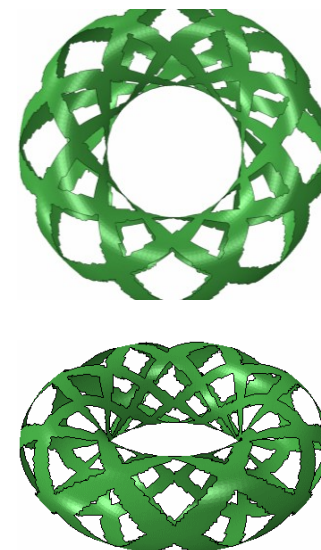
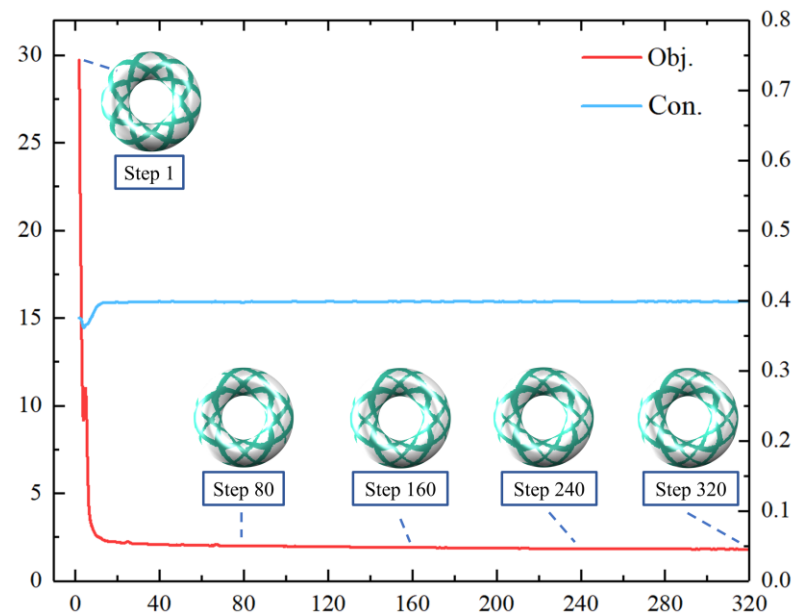


## 共形参数化与初始布局



曲面裁剪与TDF映射

## 迭代历史



## 最终设计

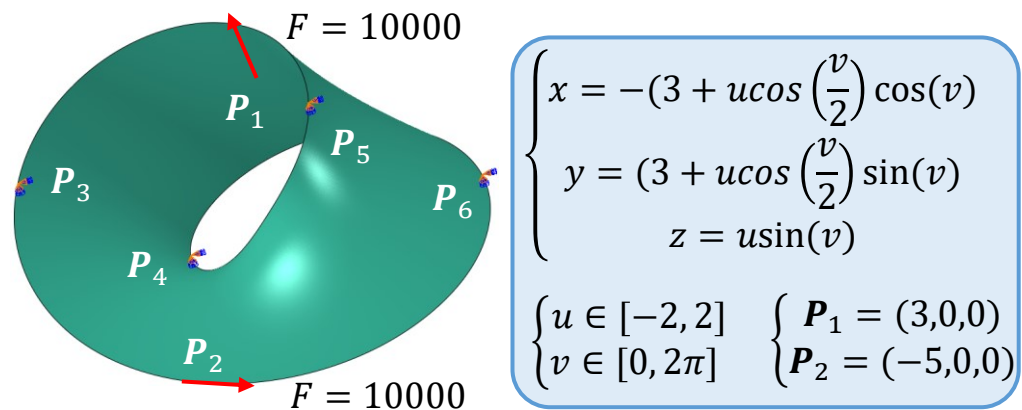


64组件 >>> 448 设计变量

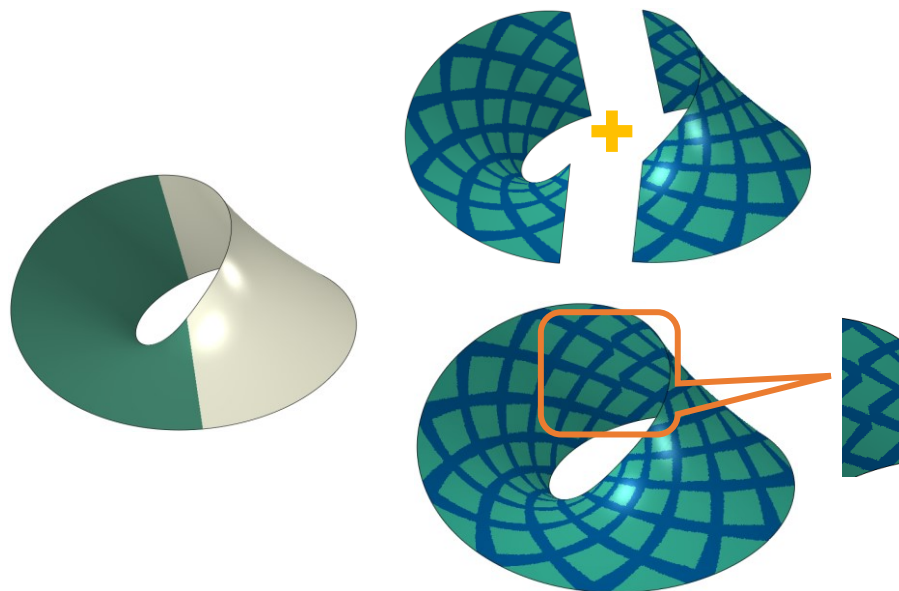


# 验证算例3: Möbius面算例

## □ 边界条件与几何尺寸

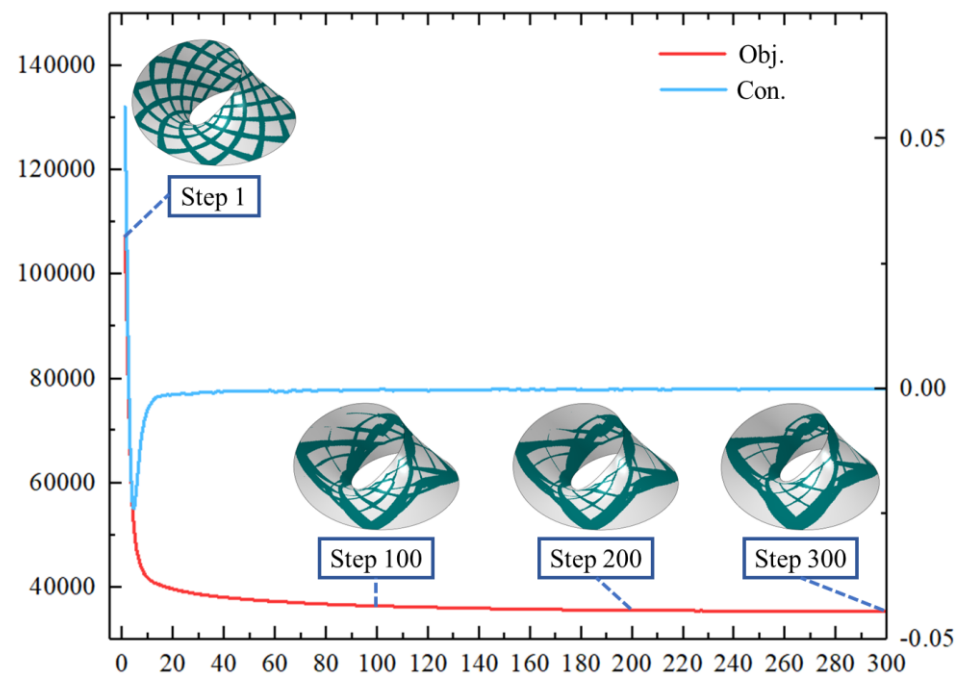


## □ 曲面分割与组件布局

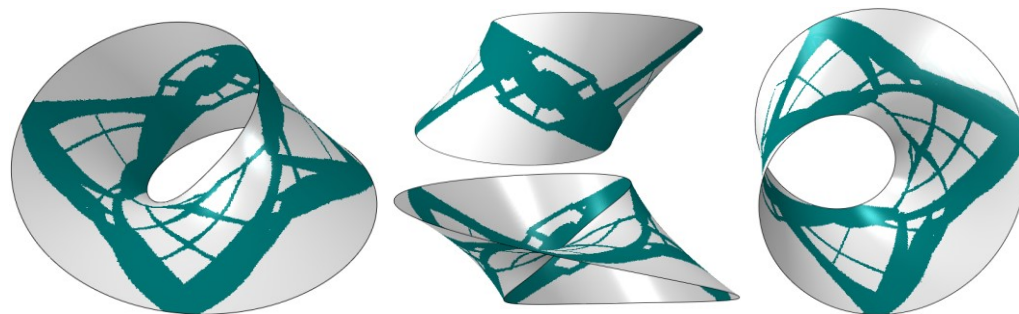


非定向曲面的分割与组件布局

## □ 迭代历史



## □ 最终设计

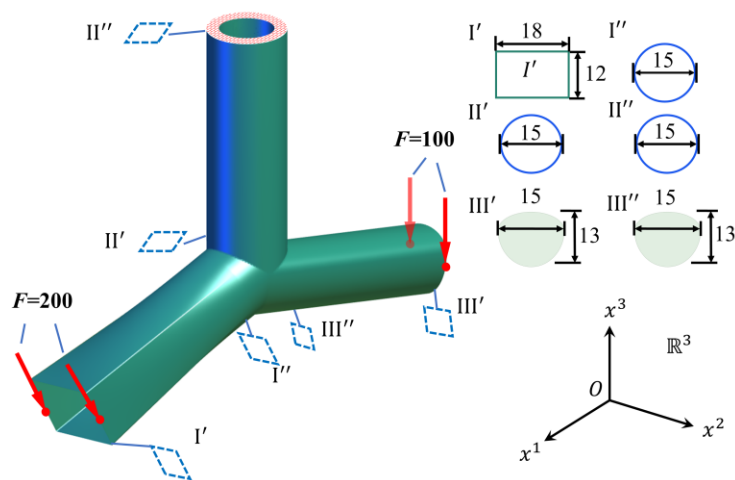


72组件 >>> 504 设计变量

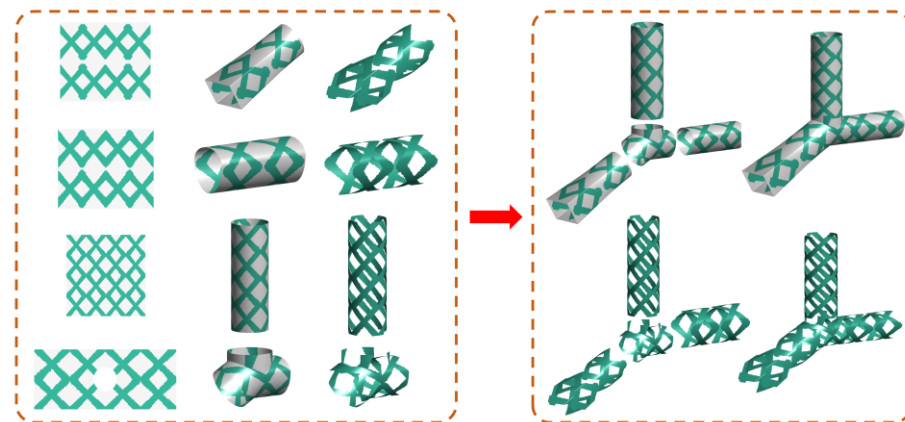


# 验证算例4：三通管道算例

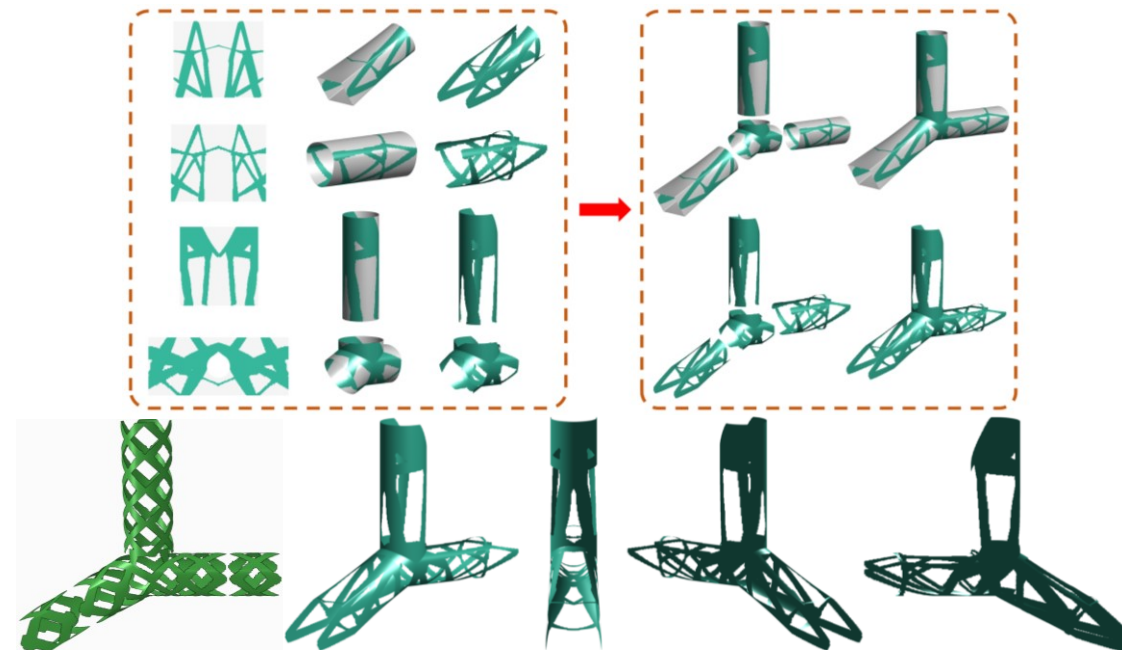
## □ 边界条件与几何尺寸



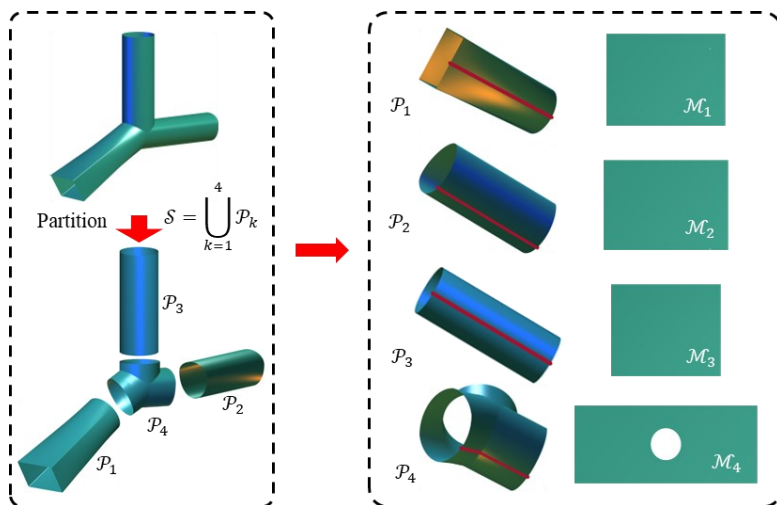
## □ 初始布局



## □ 结构演化与最终设计



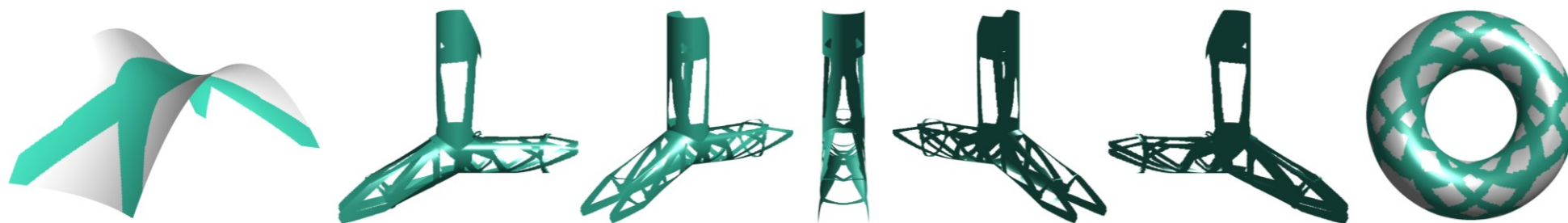
## □ 几何分割与参数化



全局参数化由分片参数化的组合实现

# 研究小结

- ✓ 提出了一种基于壳体模型的薄壁结构拓扑优化方法
- ✓ 结合了移动可变形组件法、共形参数化、曲面切割和多面片拼接技术
- ✓ 基于网格模型描述，实现了与三维扫描技术、商业CAE软件的无缝衔接
- ✓ 设计变量少、收敛迭代快、设计结果边界清晰、可处理复杂曲面薄壁结构

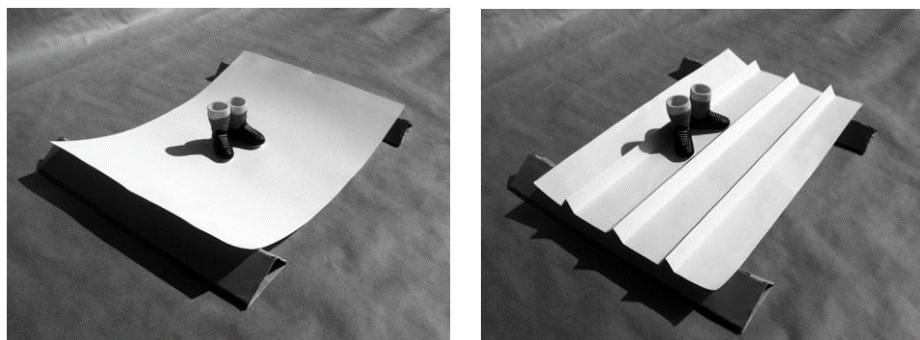


- 优化层面：其他力学问题（特征值、动力学、大变形）
- 其他物理场（声光电磁热）其他应用领域（建筑、可穿戴电子器件）
- 计算层面：曲面域相关数值计算...

## 工作 2 曲壳形貌设计 (第四章)

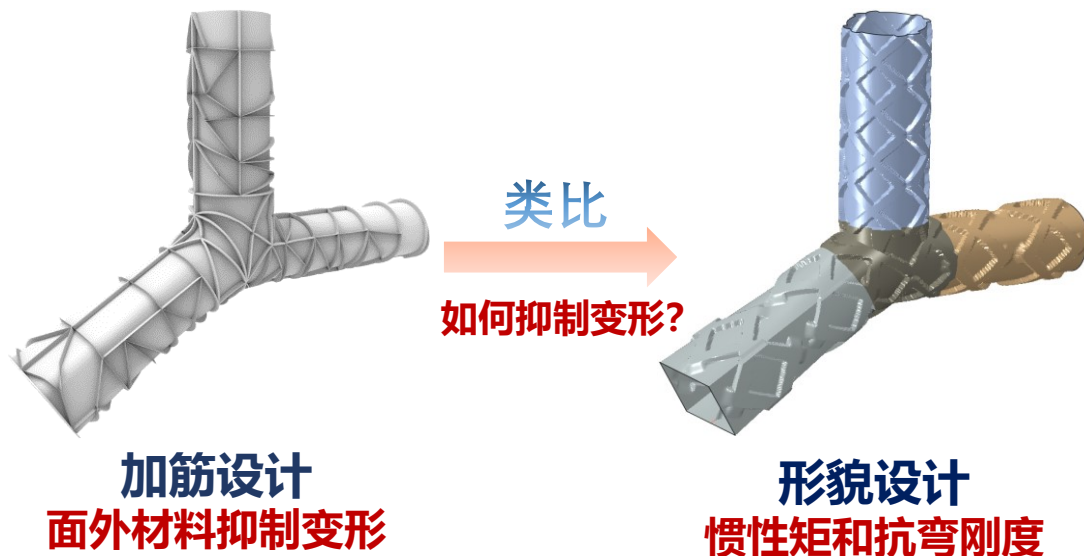
# 研究动机与问题描述

## 想法起源

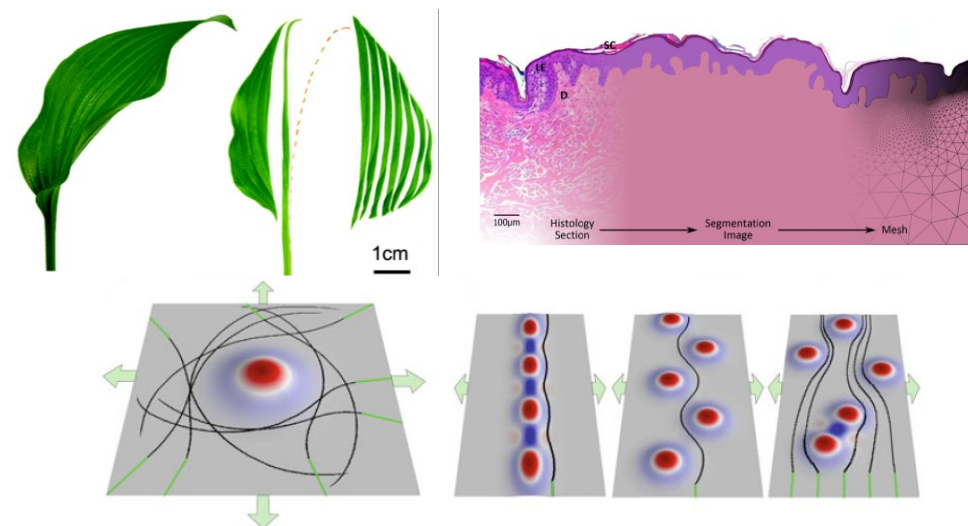


纸桥结构凹凸不平的**形貌**可大幅提升刚度

## 机理解释



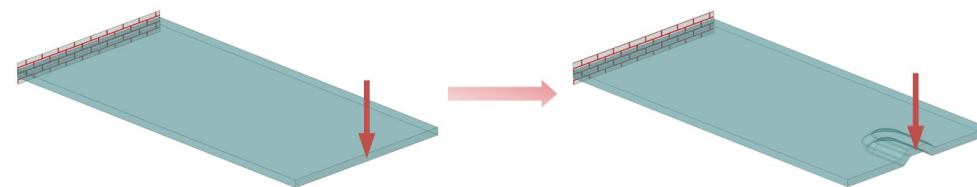
## 形貌研究现状



Liang et. al., *PNAS*, 2011; Mitchell et. al., *NM*, 2016

现有的形貌研究主要**集中于正问题**  
(形貌生成的解读、生物薄膜裂纹扩展的探索)

## 引出问题

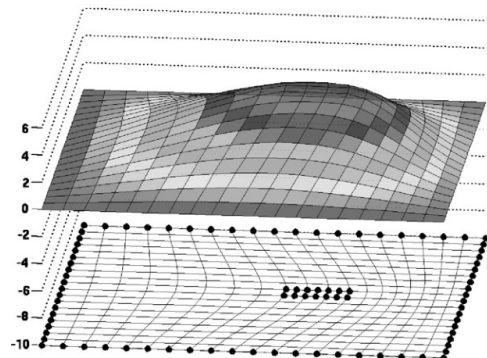
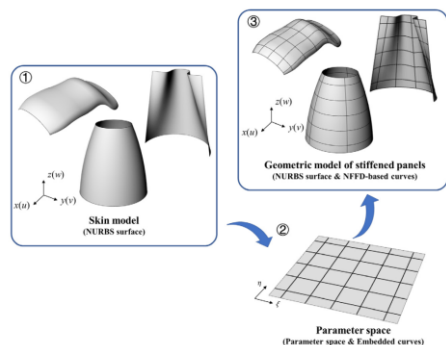
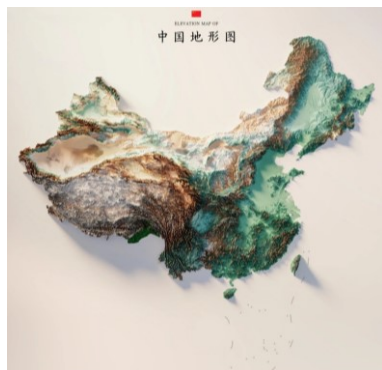


反问题：如何设计**形貌变化**提升壳体结构的性能？



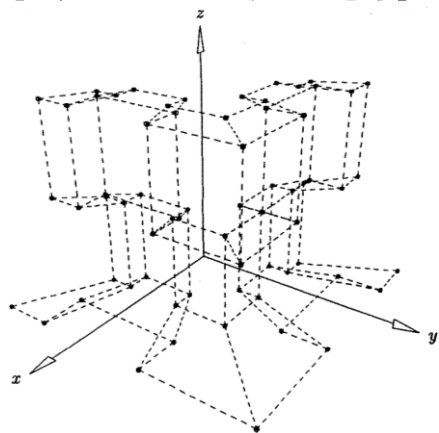
# 研究面临的难题

## □ 难题一：如何对复杂的形貌进行参数化描述

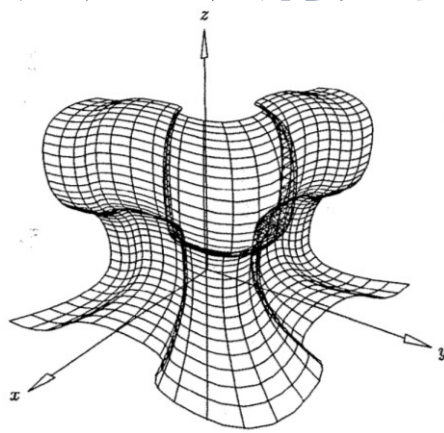


- 复杂形貌的描述需要考虑**边界形状**和**高度变化**
- 直接采用CAD工具可能需**多次重构参数描述**（迭代）才能表达形貌细节
- 基于节点描述需要**Morphing算子**来正则化网格且**设计变量数目巨大**

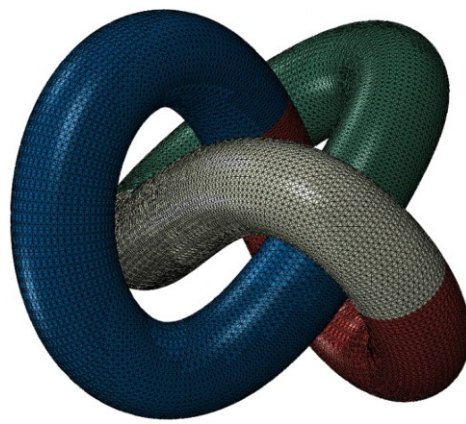
## □ 难题二：如何推广应用于具有复杂中曲面的壳体结构



样条模型 (Bezier, NURBS, T-spline...)



网格模型 (Triangle, Tetra, Hexahedral...)

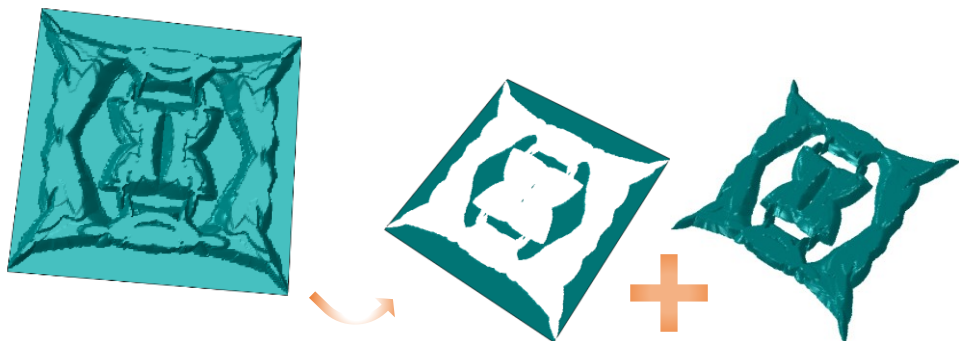


## □ 难题三：如何构建优化列式并进行灵敏度分析，使得设计进程自动化？

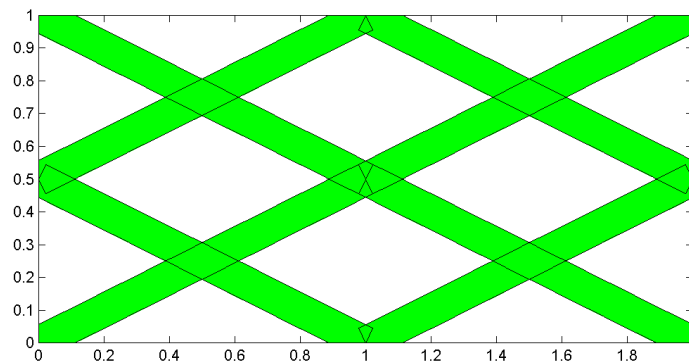


# 采用样条组件进行平面形貌的刻画

## □ 难题一：如何对复杂的形貌进行参数化描述



具有复杂形貌的结构可分解为无/有扰动区域

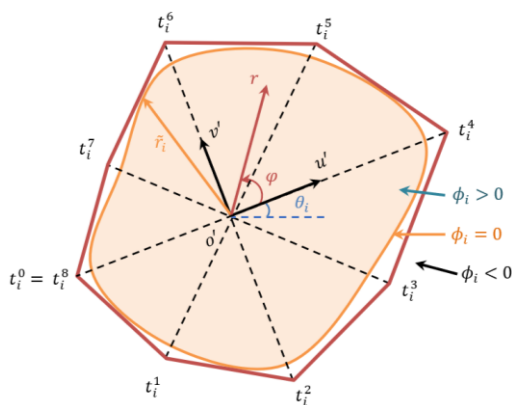


移动可变形组件法

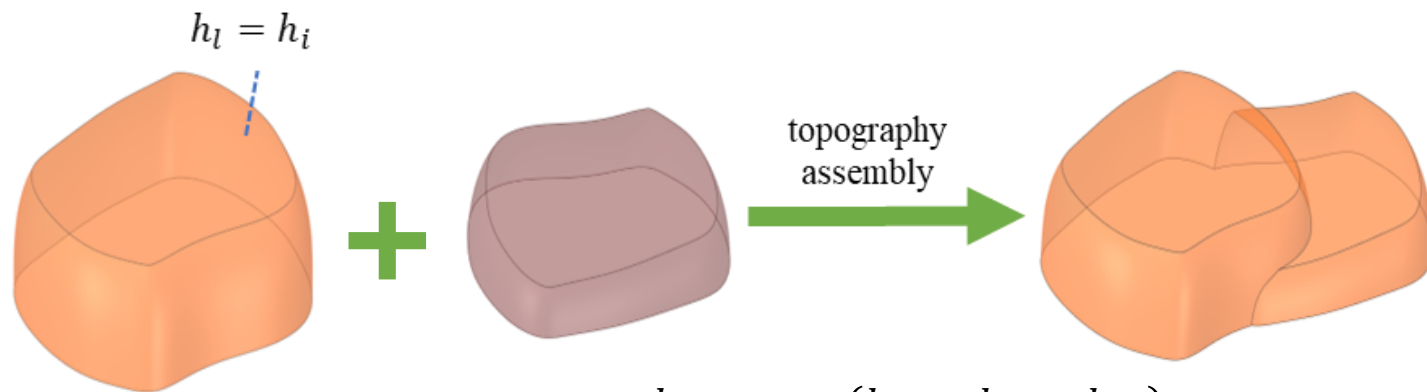
- 显式描述几何
- 设计变量少
- 边界刻画清晰
- 直接控制形貌特征

设计形貌需要解答“在哪些区域进行形貌扰动”，包含设计拓扑的需求，因此采用移动可变形组件法

## □ 样条形貌组件构建技术



样条工具构造拓扑描述函数  
刻画形貌组件边缘

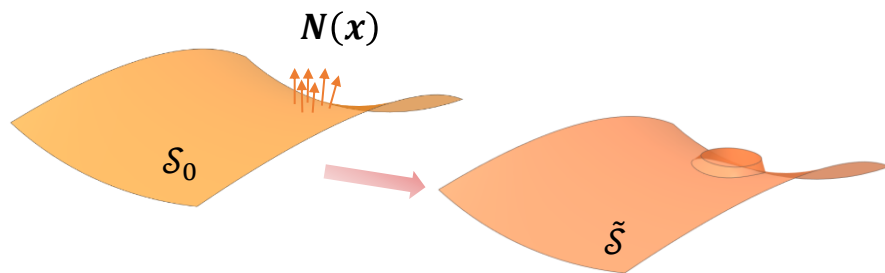


扰动高度设置为设计变量

$$h_{\mathcal{M}} = \max(h_1, \dots, h_i, \dots, h_{nc})$$
  
基于凝聚函数的形貌装配

# 基于共形参数化的嵌入式形貌描述

## □ 难题二：如何推广应用于具有复杂中曲面的壳体结构

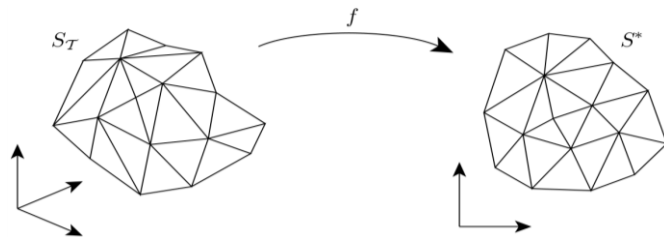


沿法向扰动中曲面得到形貌

## □ 共形参数化技术

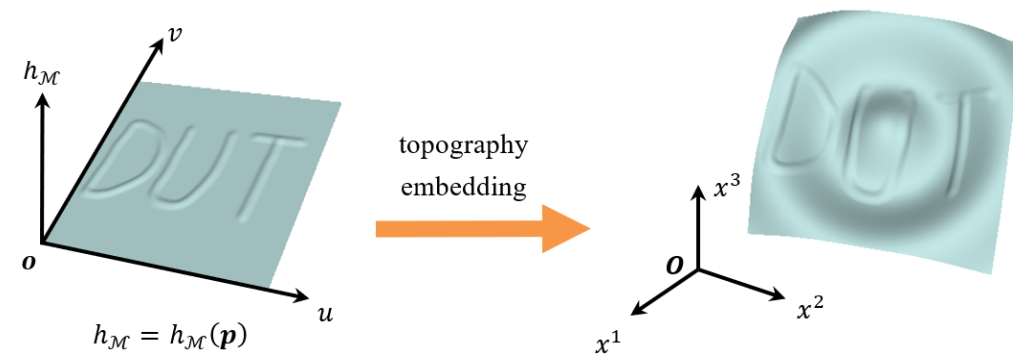
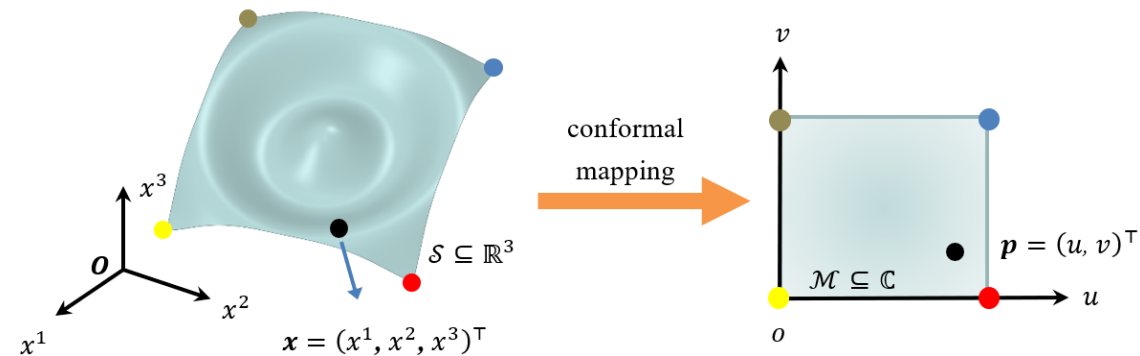
$$\begin{aligned} f &= f_2 \circ f_1 \\ \begin{cases} \Delta_S f_1 = 0, \text{ in } S \\ f_1(\partial S) = \partial D \end{cases} & \quad \begin{cases} \nabla \cdot (A(\nabla u)) = 0, \text{ in } \mathcal{D} \\ \nabla \cdot (A(\nabla v)) = 0, \text{ in } \mathcal{D} \\ f_2(\partial \mathcal{D}) = \partial M, \end{cases} \end{aligned}$$

通过求解几何PDE构造共形映射



基于网格的保角降维技术

Choi et. al., *JSC*, 65.3 (2015): 1065-1090



**Step 1: 共形映射的计算**

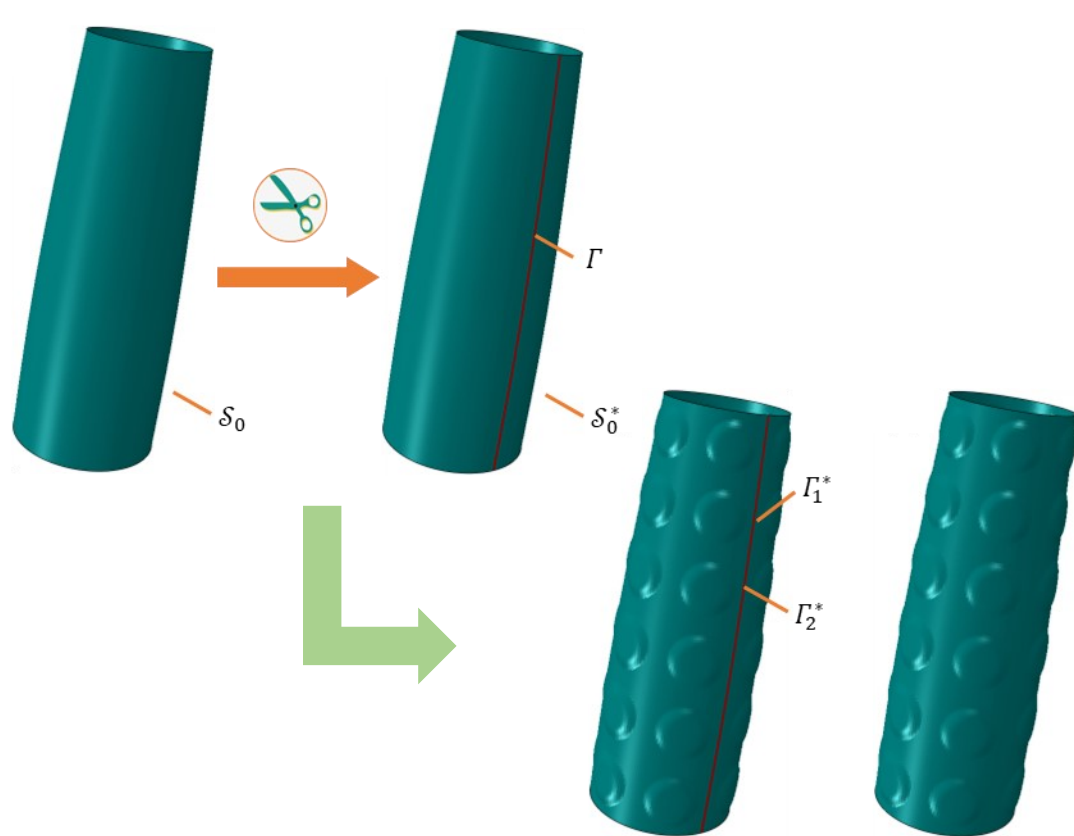
**Step 2: 在平面内构建形貌函数**

**Step 3: 基于逆映射将平面形貌嵌入曲面**

单连通零亏格开曲面的嵌入式形貌描述

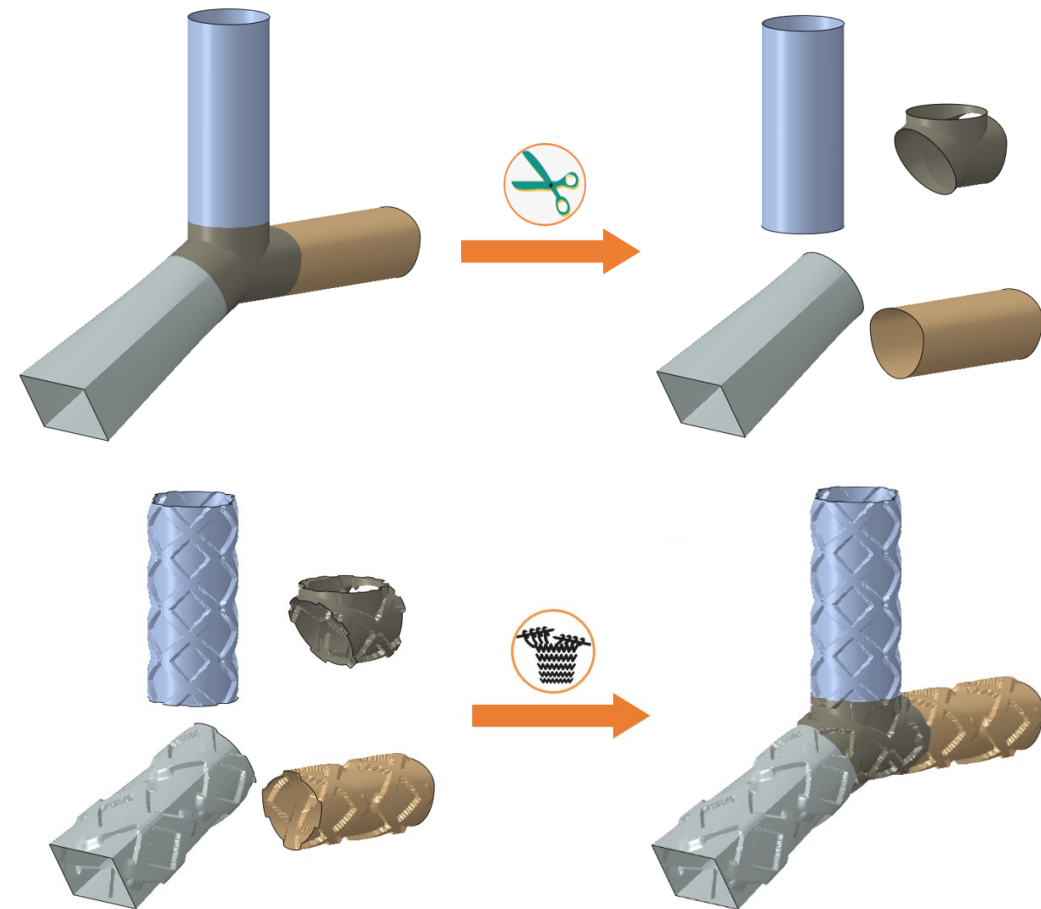
# 面向复杂曲面的嵌入式形状描述

## □ 难题二：如何推广应用于具有复杂中曲面的壳体结构



$$h_{S_0}(x; D) = \begin{cases} h_{S_0^*}(x^*; D), & \text{if } x \in S_0 \setminus \Gamma, \\ \max(h_{S_0^*}(x_1^*; D), h_{S_0^*}(x_2^*; D)), & \text{if } x \in \Gamma. \end{cases}$$

基于切割操作的形貌描述

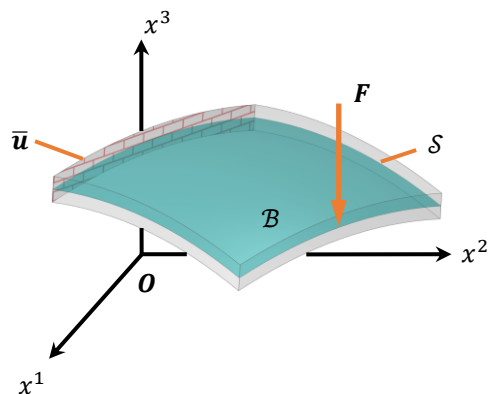


$$h_{S_0}(x; D) = \max(h_{P_1}^S, h_{P_2}^S, \dots, h_{P_{ns}}^S)$$

基于多面片拼接技术的形貌描述

# 优化列式与灵敏度分析

## □ 难题三：如何构建优化列式并进行灵敏度分析，使得设计进程自动化？



几何特征：设计域随设计变动

$$\mathcal{B}(\mathbf{D}, t) = \mathcal{S}(\mathbf{D}) \times [-\bar{t}/2, \bar{t}/2]$$

$$\partial \mathcal{B}(\mathbf{D}, t) = \bigcup_{i=1}^3 \Gamma_i$$

$$\begin{cases} \Gamma_1 = \mathcal{B}(\mathbf{D}, t) \big|_{t=\bar{t}/2} \\ \Gamma_2 = \mathcal{B}(\mathbf{D}, t) \big|_{t=-\bar{t}/2} \\ \Gamma_3 = \partial \mathcal{S}(\mathbf{D}) \times [-\bar{t}/2, \bar{t}/2] \end{cases}$$

Find  $\mathbf{D}, \mathbf{U}(\mathbf{x}; \mathbf{D})$  参数域中的形貌组件几何信息

$$\text{Minimize } C = C(\mathbf{U}(\mathbf{x}; \mathbf{D}), \mathbf{D}) = \int_{\mathcal{B}(\mathbf{D})} (\mathbf{F}(\mathbf{D}) \cdot \mathbf{U}(\mathbf{x}; \mathbf{D})) dV$$

S. t.

$$\begin{aligned} & \int_{\mathcal{B}(\mathbf{D})} W_C(\mathbf{U}(\mathbf{x}; \mathbf{D}), \mathbf{V}(\mathbf{x}; \mathbf{D})) dV \\ & = \int_{\mathcal{B}(\mathbf{D})} (\mathbf{F}(\mathbf{D}) \cdot \mathbf{V}(\mathbf{x}; \mathbf{D})) dV, \forall \mathbf{V}(\mathbf{x}; \mathbf{D}) \in \mathcal{U}_{ad} \end{aligned}$$

$$\mathbf{U}(\mathbf{x}; \mathbf{D}) \in \mathcal{U}, \mathbf{D} \in \mathcal{U}_D$$

$$\begin{aligned} \frac{DC}{Dd} &= 2 \int_{\mathcal{B}(d)} W_C(\mathbf{U}'(\mathbf{x}; d), \mathbf{U}(\mathbf{x}; d)) dV + \int_{\partial \mathcal{B}(d)} W_C V_n dS \\ &= 2 \int_{\mathcal{B}(d)} \frac{d}{dd} \mathbf{F}(d) \cdot \mathbf{U}(\mathbf{x}; d) dV + \int_{\partial \mathcal{B}(d)} (2W_F - W_C) V_n dS \end{aligned}$$

形状灵敏度

$$\begin{aligned} \mathbf{x}'_1 &= \mathbf{x}' + \frac{\bar{t}}{2} \mathbf{N}(\mathbf{x}', \mathcal{S}(d)) \\ \mathbf{v}_1 &= \frac{D}{Dd} \mathbf{x}'_1(\mathbf{x}; d) = \frac{\partial}{\partial d} h_{\mathcal{S}}(\mathbf{x}; d) \cdot \mathbf{N}(\mathbf{x}, \mathcal{S}_0) + \frac{\bar{t}}{2} \frac{\partial \mathbf{N}(\mathbf{x}', \mathcal{S}(d))}{\partial d} \\ V_n^1 &= \mathbf{v}_1 \cdot \mathbf{N}(\mathbf{x}'_1, \Gamma_1) \end{aligned}$$

速度场推导

$$\begin{aligned} \frac{DC}{Dd} &= 2 \sum_{e=1}^{n_e} \left( \frac{d}{dd} \mathbf{F}(d) \cdot \mathbf{U}(\mathbf{x}; d) \right)_e V_e \\ &+ \sum_{e=1}^{n_e} \left( ((2W_F - W_C) V_n)_e^{outer} + ((2W_F - W_C) V_n)_e^{inner} \right) A_e \end{aligned}$$

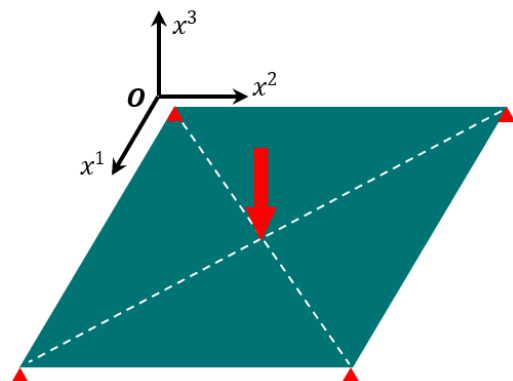
数值求解

D. Chapelle, K.-J. Bathe, *The finite element analysis of shells-fundamentals*, Springer Science & Business Media, 2010.

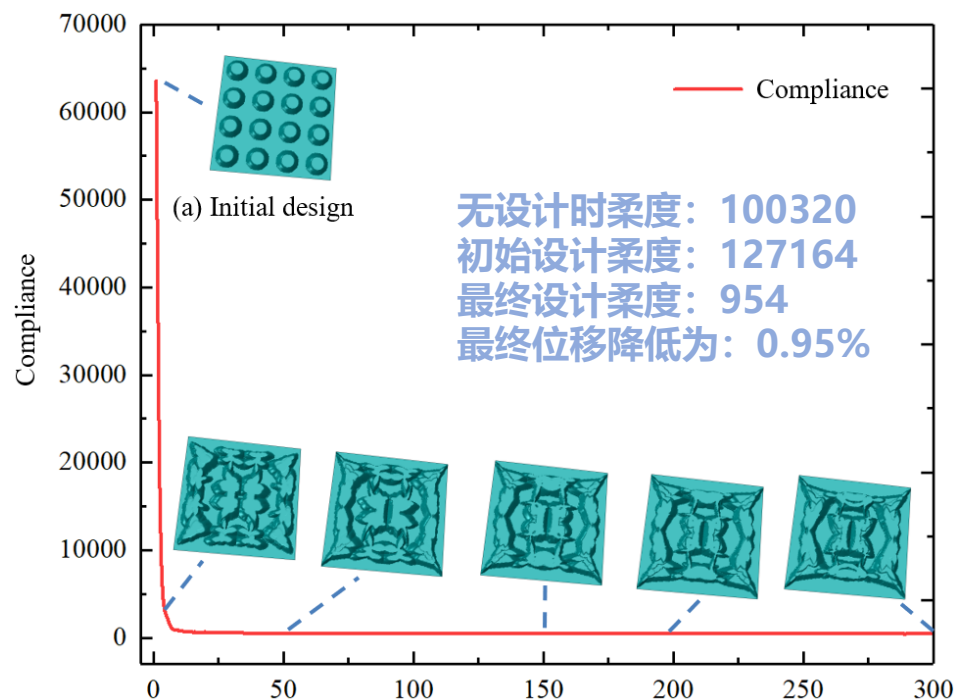
根据灵敏度分析，得到设计变量的演化方向，并且进一步调整结构中组件的尺寸与布局

# 数值实验1：平直设计域（方板）形貌优化

## 算例一：方板

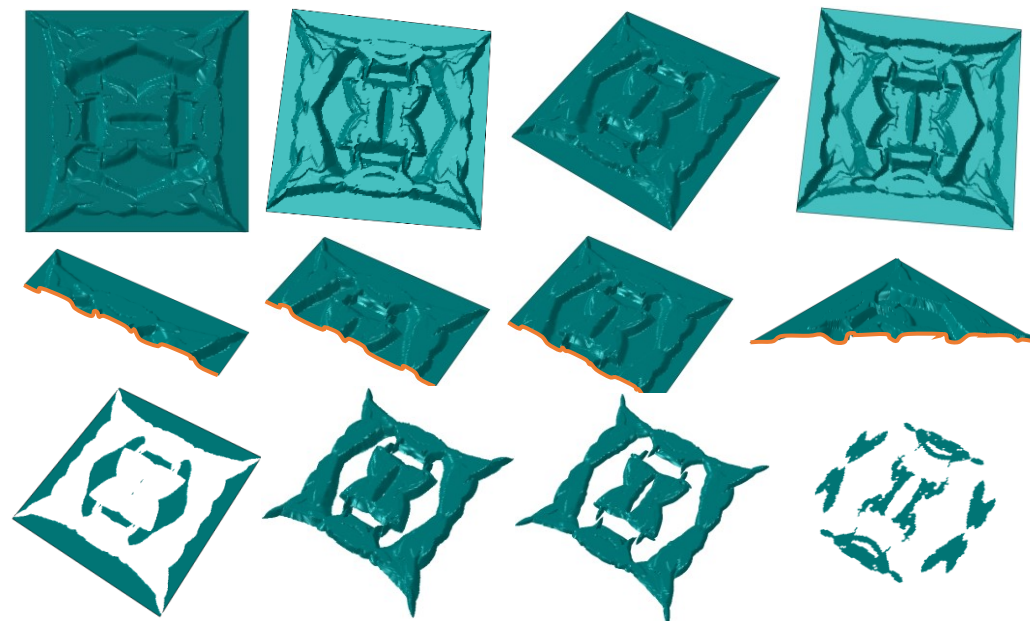


边长: 80  
厚度: 0.2  
弹性模量: 1  
泊松比: 0.3  
集中力大小: 1  
集中力方向:  $-x^3$   
形貌高度上限: 4



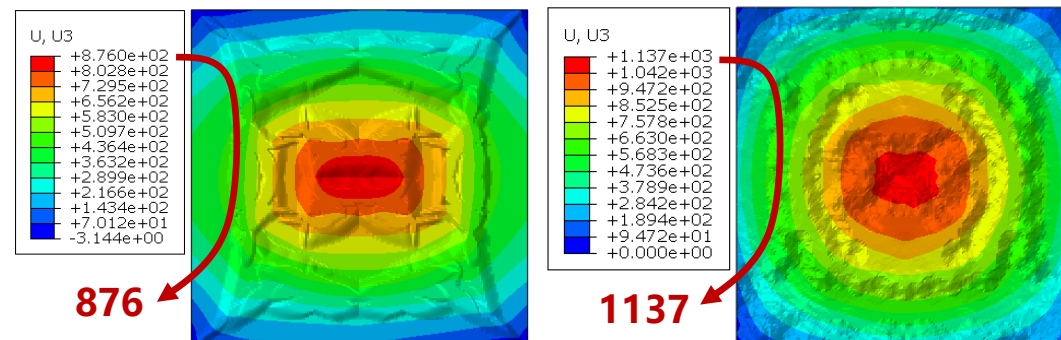
此为迭代历史，大约50步即可迭代出最终轮廓

## 最终设计展示



丰富的形貌细节是提升结构性能的关键!

## 商软对比 (导入OptiStruct统一求解位移场)



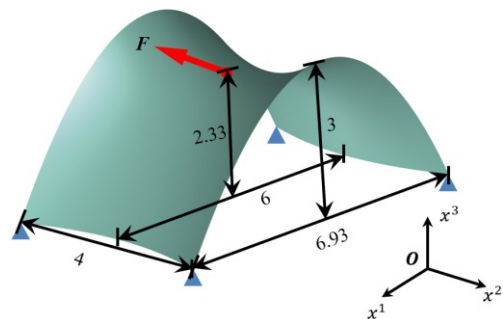
本文设计结果

OptiStruct 设计结果

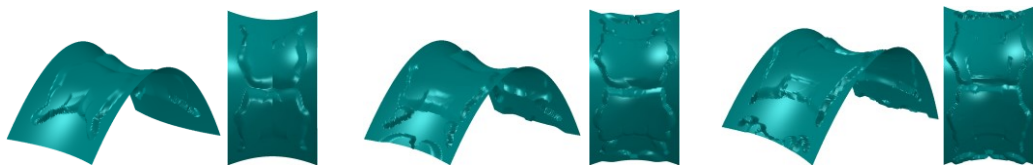
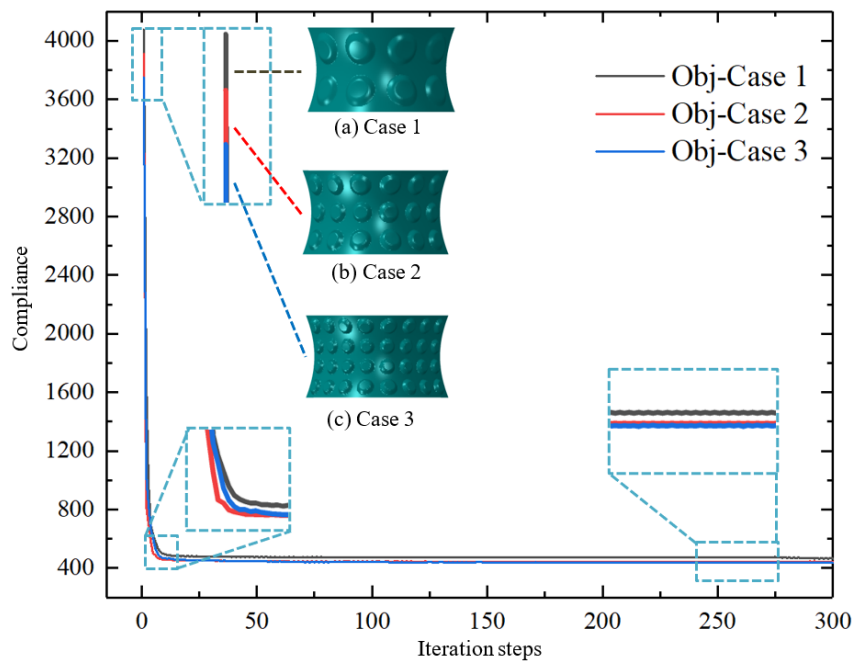


# 数值实验2：基于共形参数化技术的曲壳形貌设计

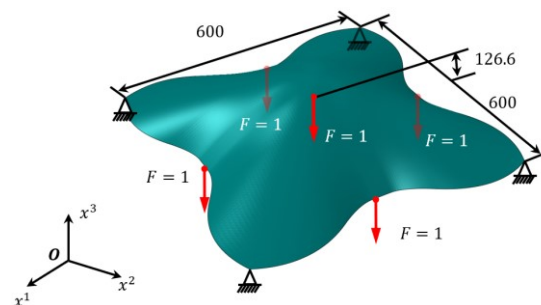
## 算例二：鞍面



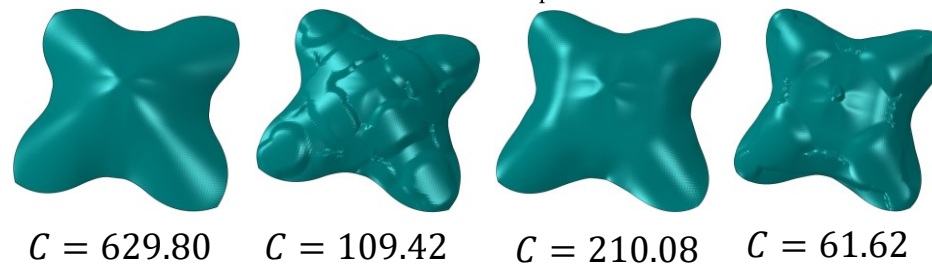
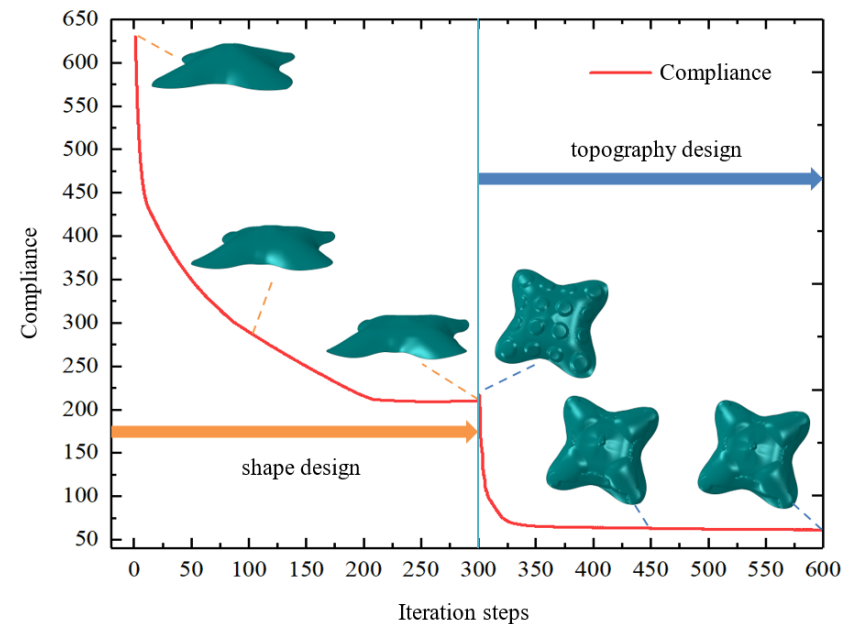
厚度: 0.02  
弹性模量: 1  
泊松比: 0.3  
集中力方向:  $-x^2$   
形貌高度上限: 0.2



## 算例三：穹顶（形状+形貌联合优化）

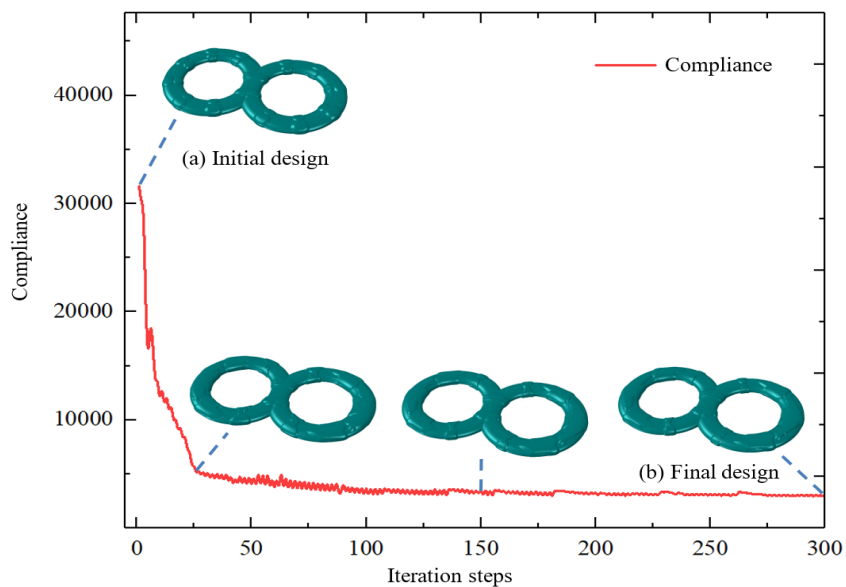
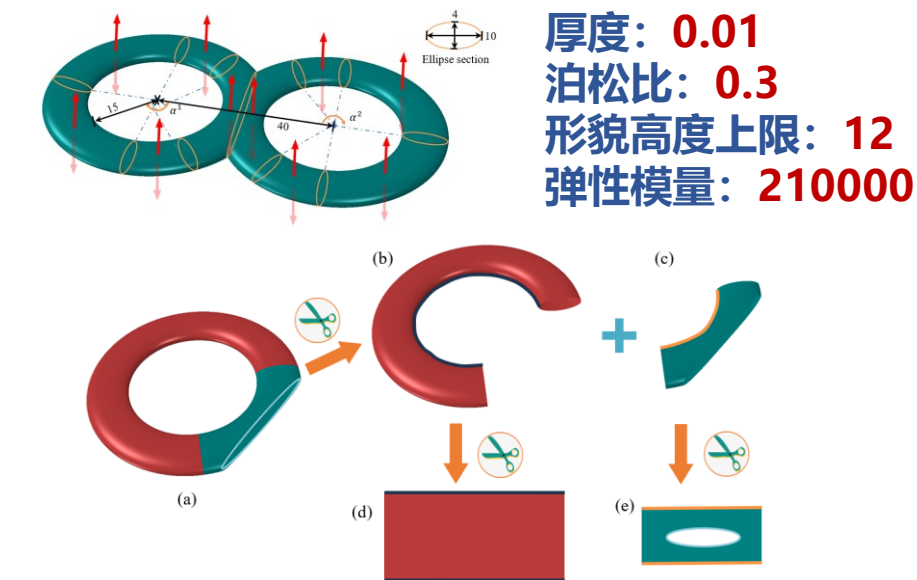


厚度: 3  
弹性模量: 1  
泊松比: 0.3  
集中力方向:  $-x^3$   
形貌高度上限: 12

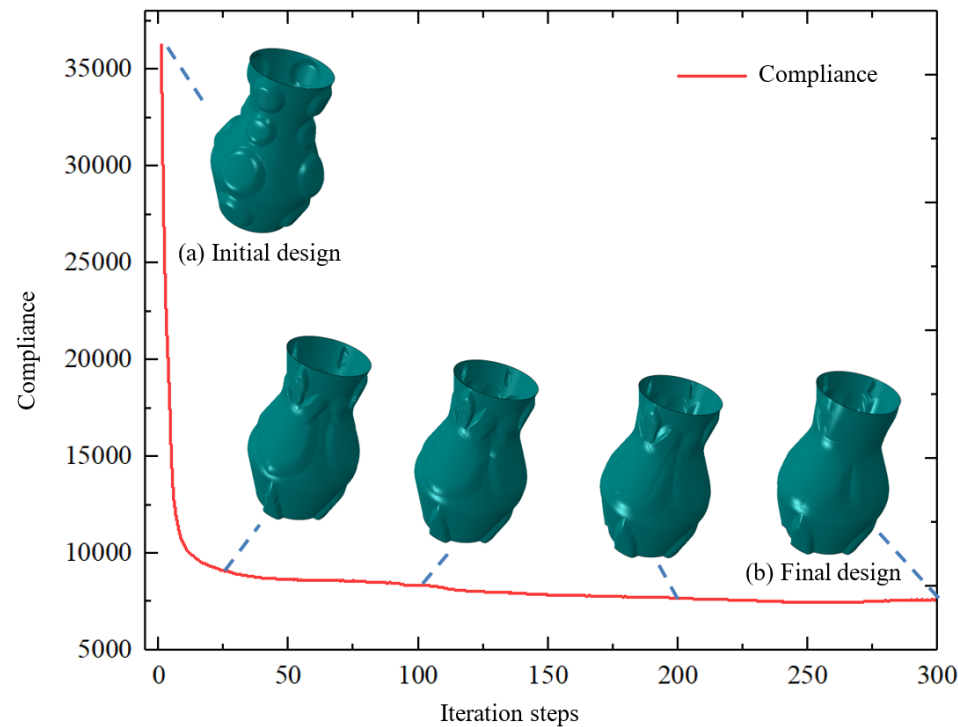
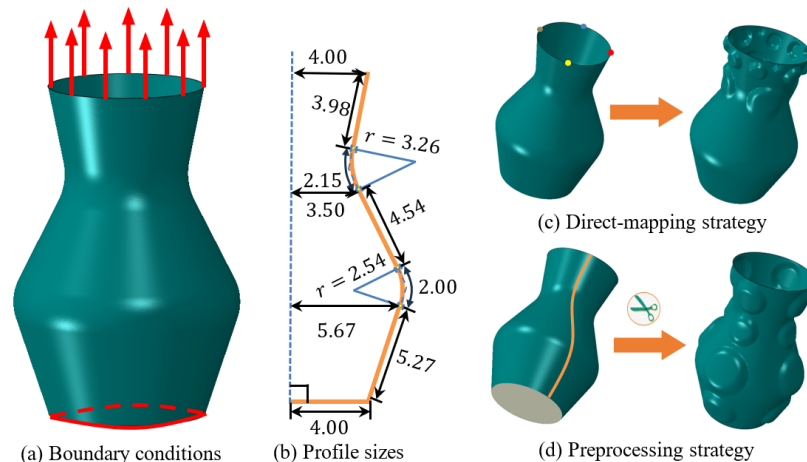


# 数值实验3：一般复杂曲面壳体结构形貌优化

## 算例四：八字环面

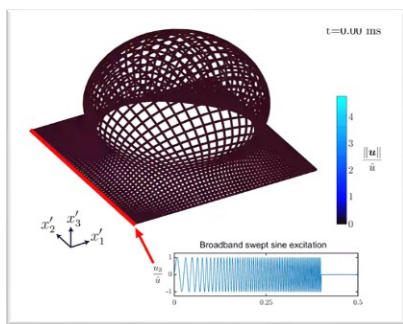


## 算例五：花瓶曲面

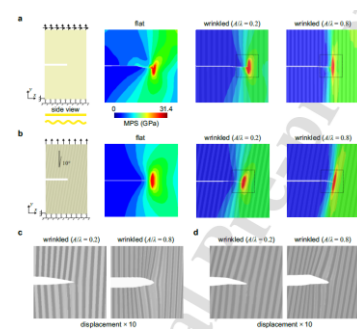
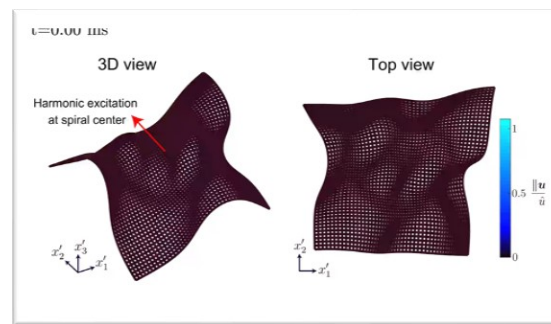


# 研究小结

- ✓ 所提方法可有效处理复杂曲面形貌设计问题
- ✓ 得益于MMC方法，本方法可采用较少的设计变量描述精细的形貌特征
- ✓ 集成的曲面切割操作与多面片拼接技术有效地处理了单一映射带来的畸变问题



设计形貌控制波导路径



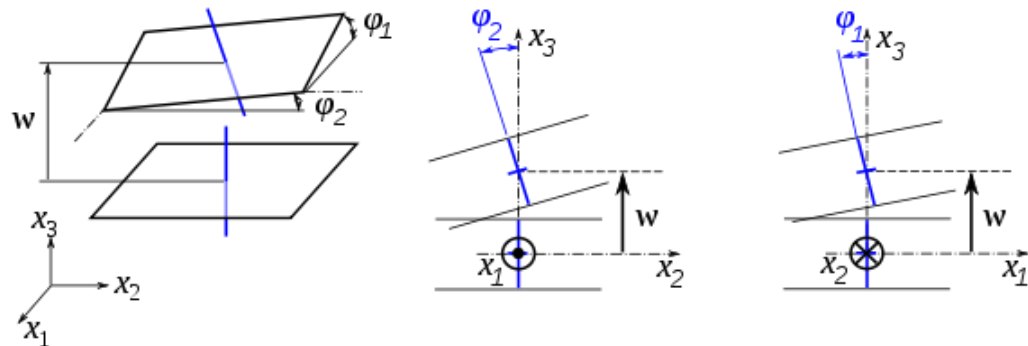
控制断裂路径



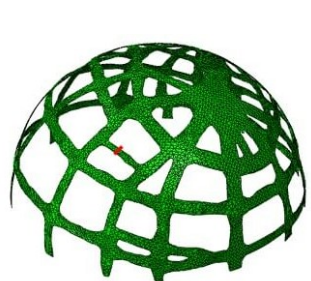
- 将形貌扰动作为一种结构调控手段，应用于不同场景（壳体断裂、生物薄膜）
- 考虑制造细节（Metal forming: plasticity, contact, fracture, impact...）
- 特殊的扰动类型（lattice类、初始缺陷类（折痕剪痕））

## 工作 3 曲体拓扑优化 (第五章)

## 研究动机



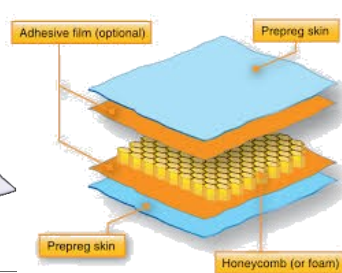
壳体模型具有较高的建模和分析效率  
但其底层的理论假设会限制优化算法的应用范围



拓扑设计



加筋设计



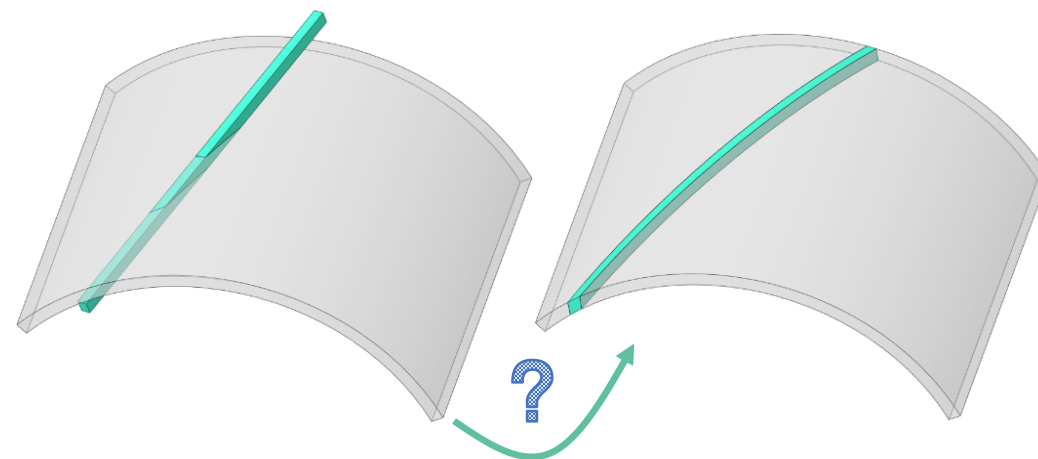
夹芯层设计

厚度方向上存在多样的几何设计需求

薄壁结构复杂性来源于其厚度变化和多样化的设计需求

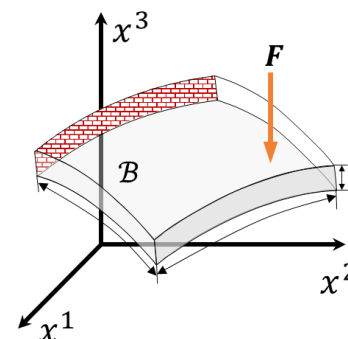
## 提取问题

挑战：平直组件难以适用沿曲面分布材料的几何约束



如何构建自然贴合曲面几何的新型实体组件  
实现多类薄壁结构设计问题的统一建模优化

## 任务拆解

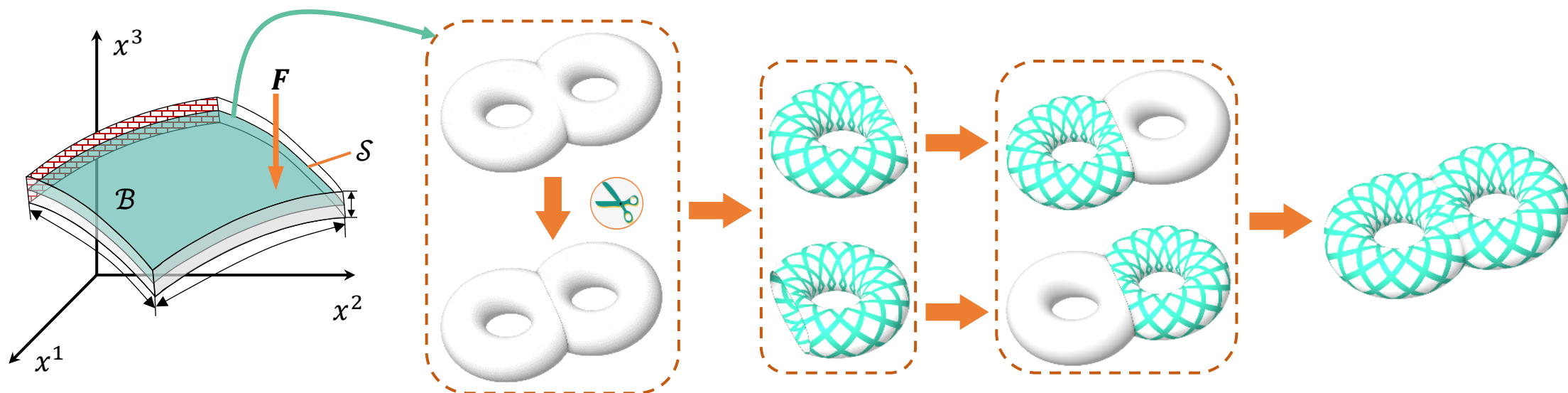


- ① 随形/贴体实体组件的构建
- ② 多样化设计的建模与描述
- ③ 优化列式、灵敏度、数值实现



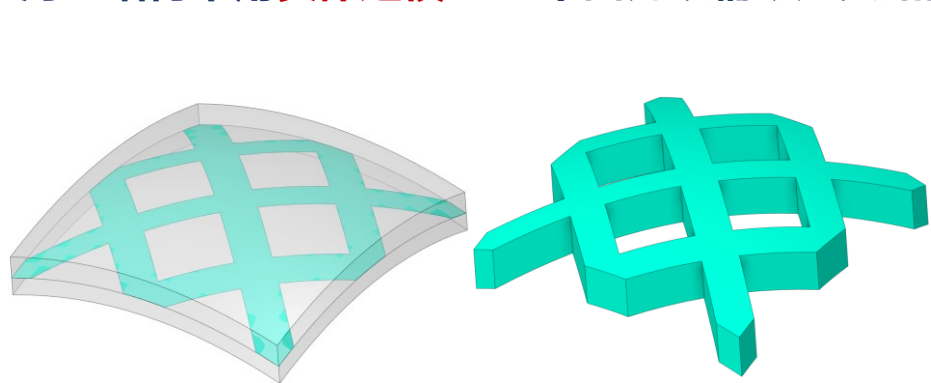
# 解决方案：基于厚度坐标的嵌入式组件

## □ 任务一：构建贴合曲面薄壁结构的随形实体组件

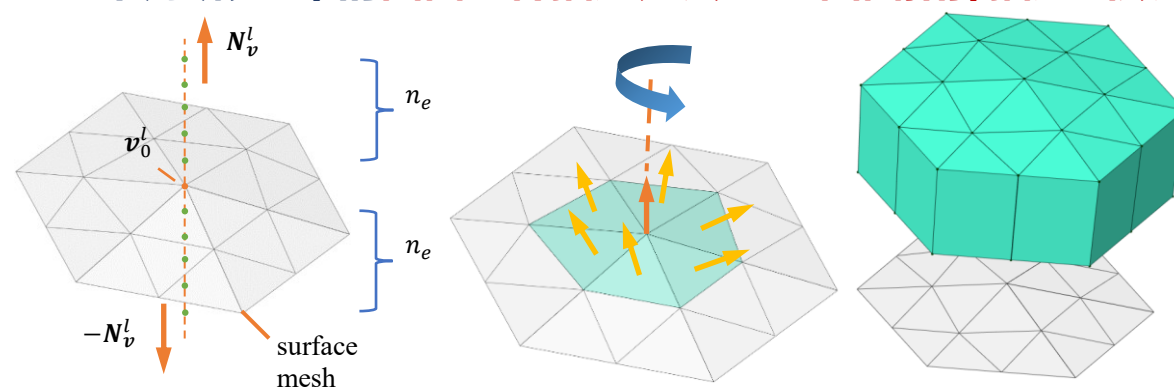


薄壁结构采用**实体建模**

中面用以辅助定义贴体组件>>>采用研究1中的**曲面组件描述方法**>>>**曲面拓扑描述函数**



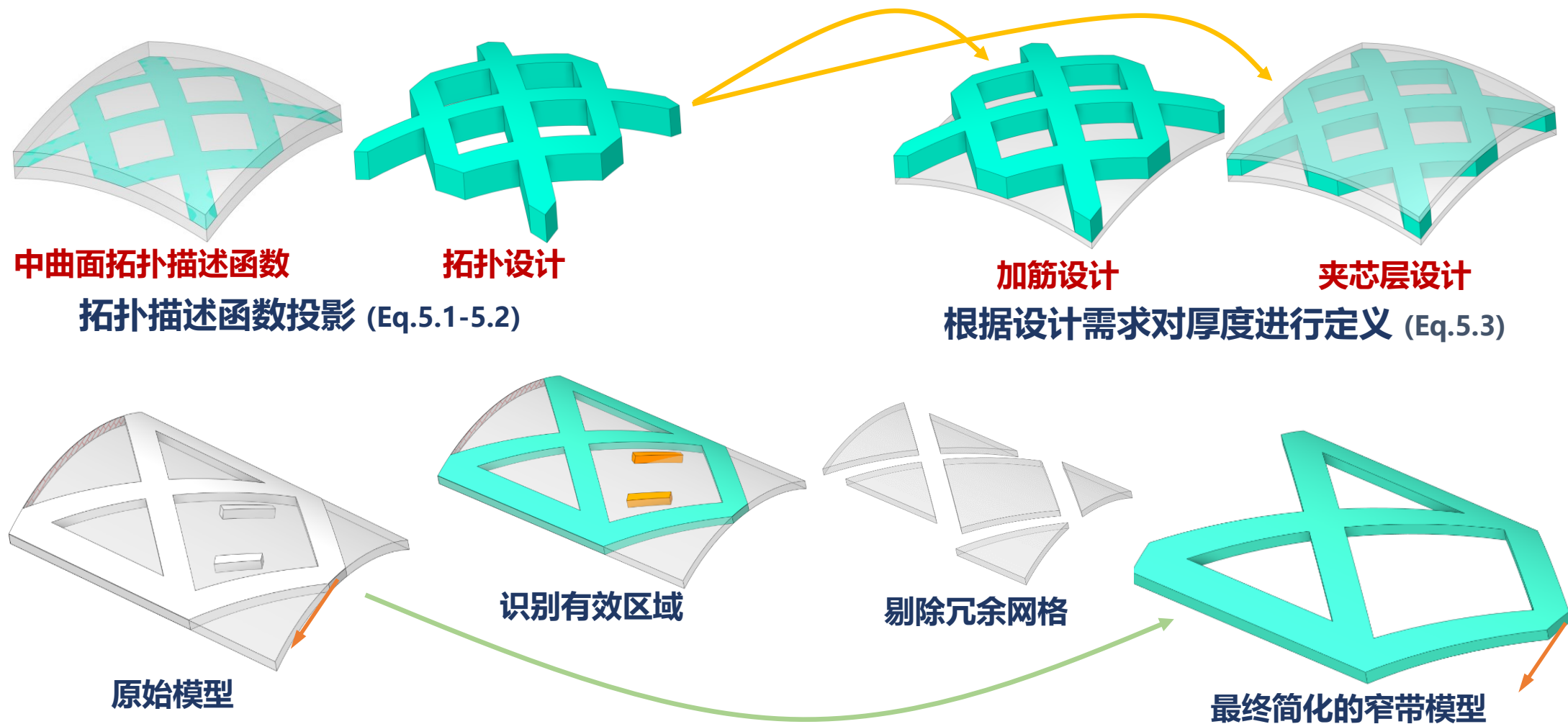
实体上任一点取中面投影点的**曲面TDF值**  
(**拓扑描述函数投影**)



**基于offset方法生成实体网格**  
(通过定义特殊的结构化网格，简化投影点的计算)

# 解决方案：基于厚度坐标的嵌入式组件

## □ 任务二：针对多样化的设计需求提出统一的建模与描述方法



基于冗余自由度删除技术获得窄带网格，提高求解效率

# 优化列式与问题求解

## 任务三：进一步构建优化列式、推导灵敏度、以及如何数值实现

### ✓ 优化列式

Find  $\mathbf{D}, \mathbf{u}(\mathbf{x})$

$$\text{Minimize } C = \int_B \rho_B^s(\mathbf{x}; \mathbf{D}) \mathbf{F} \cdot \mathbf{u} dV + \int_{\partial B} \mathbf{t} \cdot \mathbf{u} dS$$

S. t.

$$\int_B \rho_B^s(\mathbf{x}; \mathbf{D}) \mathbb{E} : \boldsymbol{\varepsilon}_{3d}(\mathbf{u}) : \boldsymbol{\varepsilon}_{3d}(\mathbf{v}) dV$$

$$= \int_B \rho_B^s(\mathbf{x}; \mathbf{D}) \mathbf{F} \cdot \mathbf{v} dV + \int_{\partial B} \mathbf{t} \cdot \mathbf{v} dS, \forall \mathbf{v} \in \mathcal{U}_{ad}^{3d},$$

$$\int_B \rho_B^s(\mathbf{x}; \mathbf{D}) dV \leq \bar{V} \int_B dV,$$

$$\mathbf{u} \in \mathcal{U}^{3d}, \mathbf{D} \in \mathcal{U}_D$$

$$\mathbf{D}_i = (x_0^i, y_0^i, \theta_i, L_i, t_i^1, t_i^2, t_i^3)^\top$$

类似二/三维 MMC 方法，区别在于设计变量为参数域几何信息

### ✓ 灵敏度分析与求解

$$I(\mathbf{u}, \mathbf{u}) = \int_B \rho_B^s(\mathbf{x}; \mathbf{D}) W(\mathbf{u}, \mathbf{u}) dV$$

$$\delta I(\mathbf{u}, \mathbf{u}) = \int_B \delta \rho_B^s(\mathbf{x}; d) W(\mathbf{u}, \mathbf{u}) + \rho_B^s(\mathbf{x}; d) \delta W(\mathbf{u}, \mathbf{u}) dA$$

一般形式的响应函数及其变分

$$\frac{\partial C}{\partial d} = - \int_B \frac{\partial \rho_B^s(\mathbf{x}; d)}{\partial d} \mathbb{E} : \boldsymbol{\varepsilon}_{3d}(\mathbf{u}) : \boldsymbol{\varepsilon}_{3d}(\mathbf{u})$$

应用于柔度的灵敏度分析中

### ✓ 数值实现与离散敏度

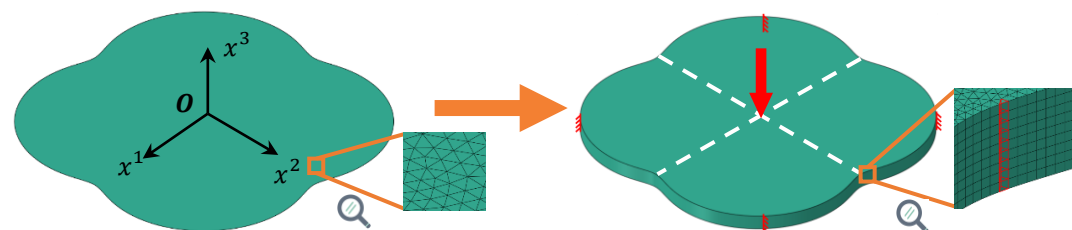
材料插值  $E_e = \frac{\sum_{i=1}^6 (\rho_B^s)_{e,i}}{6} E^s$  

$$\frac{\partial C}{\partial d} = -\mathbf{U}^\top \frac{\partial \mathbf{K}}{\partial d} \mathbf{U} = - \sum_e^{n_e} \frac{\partial E_e}{\partial d} \frac{\mathbf{U}_e^\top \mathbf{k}_0 \mathbf{U}_e}{E^s} = - \sum_e^{n_e} \frac{\partial E_e}{\partial d} \frac{C_e}{E_e}$$

核心在于单元应变能的计算和提取

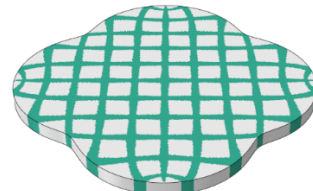
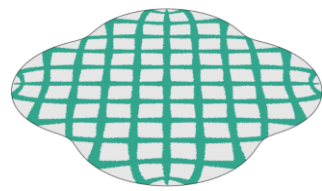
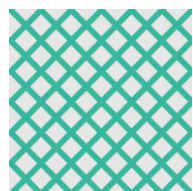
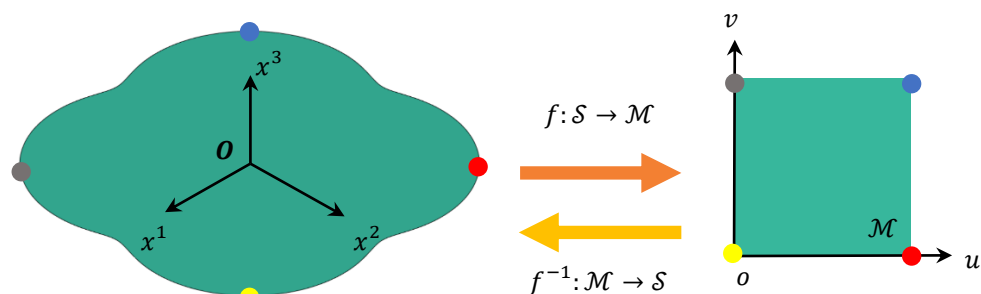
# 数值实验一：平板算例验证

## □ 网格生成与边界条件的施加

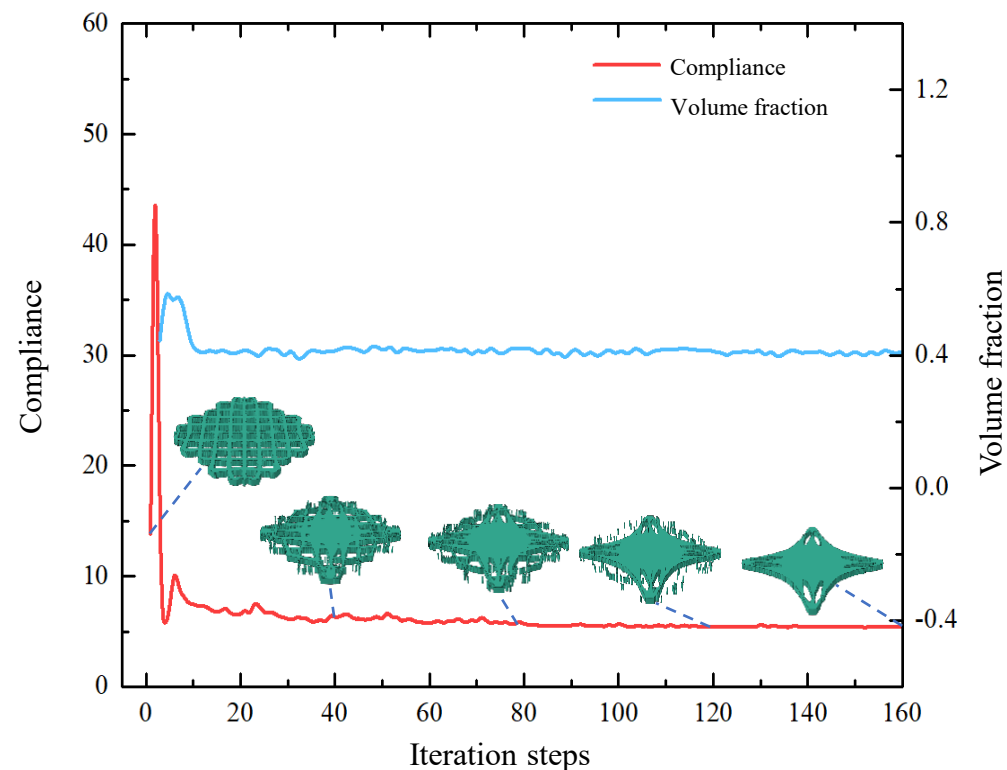


从中面生成**结构化的实体网格**

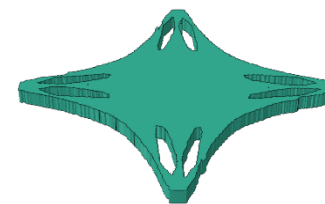
**边界条件沿厚度均匀施加**



摆线形平板结构的**映射与初始组件布局设置**



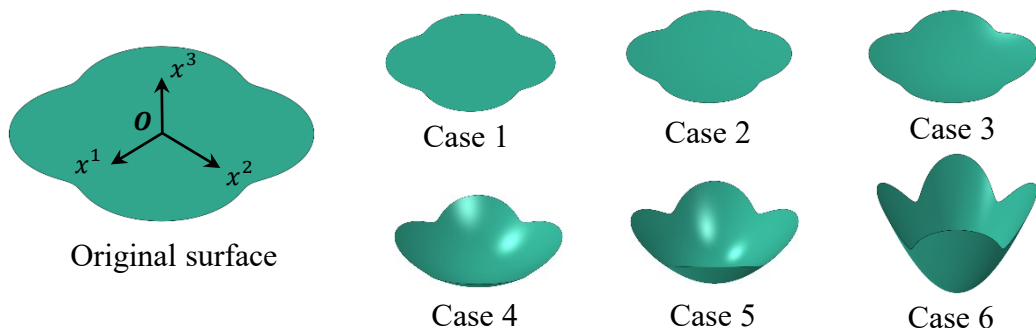
摆线形壳体算例 迭代历史及中间设计 (160步停止)



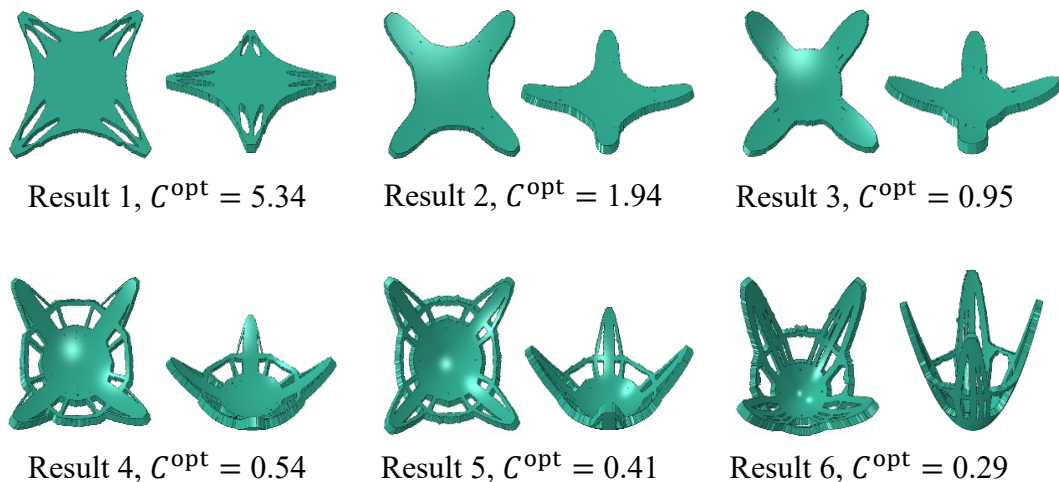
摆线形壳体算例 最终设计 (柔度=5.34)

# 数值实验二：曲体拓扑设计与算法特性测试

## 不同曲率下设计结果的测试

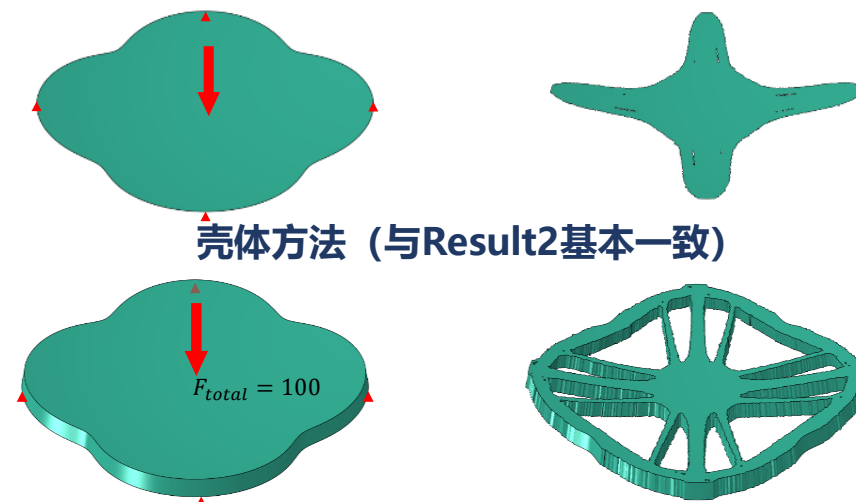


根据高度系数调整中曲面的曲率



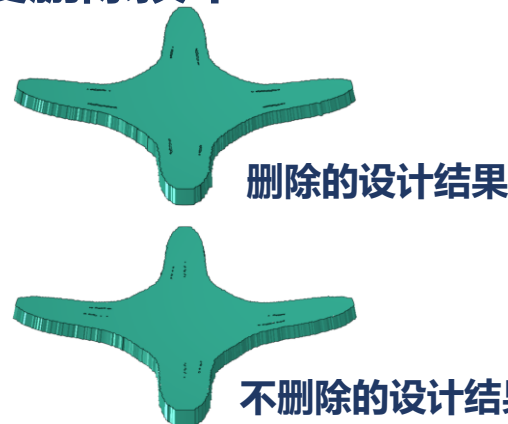
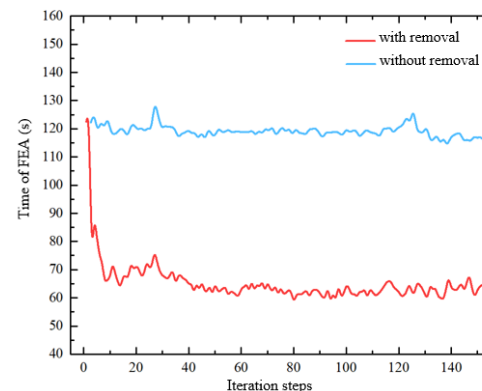
不同曲率下的最终设计结构 曲率越大，刚度越大！

## 算法独特性验证：壳体 V.S. 实体



边界条件按照点类型施加：壳体模型难以实现

## 提升效率：采用自由度删除技术



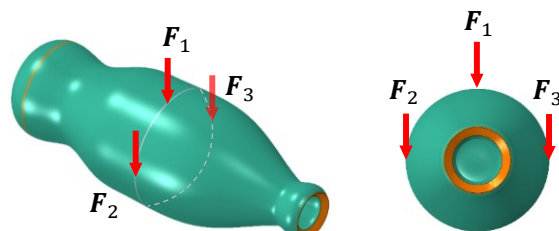
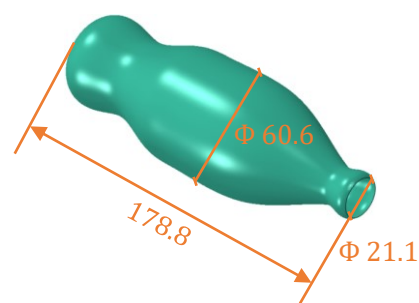
删除/不删除自由度的时间对比

平均减少44.91%的迭代时间

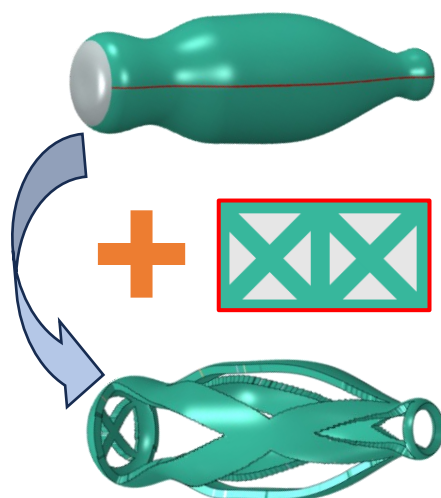


# 数值实验三：基于多面片拼接的复杂曲体结构设计

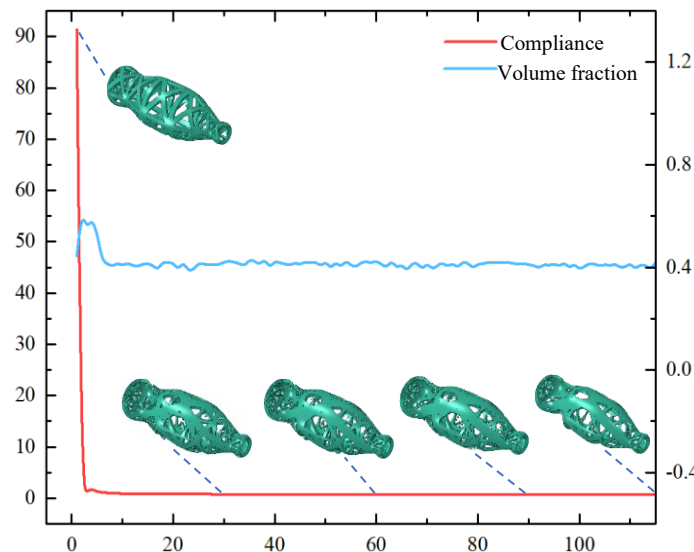
## 问题定义与拓扑设计



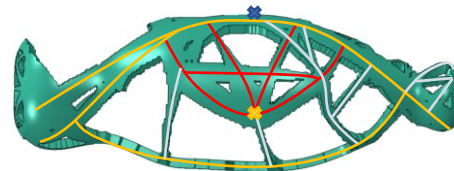
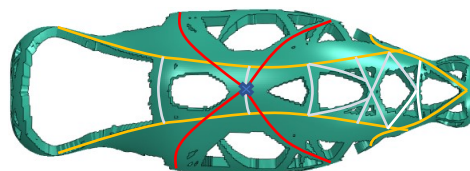
几何尺寸与边界条件



曲面分割策略

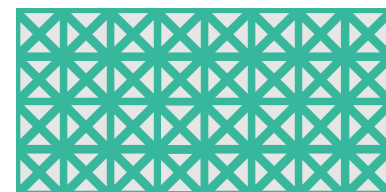


迭代历史 (迅速收敛)

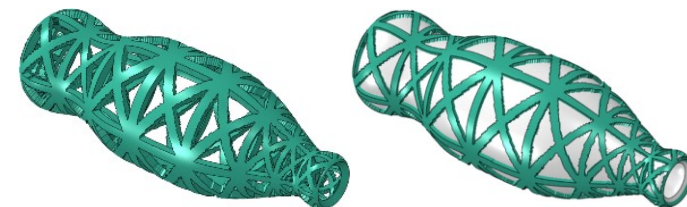


最终设计：分层式组件布局，各有分工

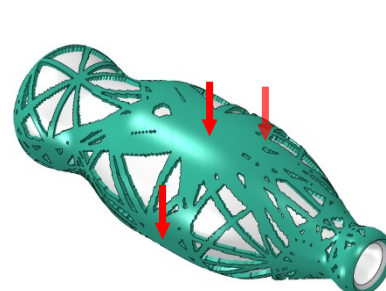
## 加筋设计



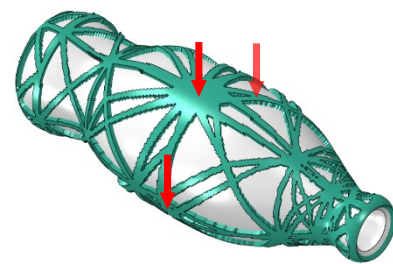
组件的参数域布局



贴体组件和进一步的贴体筋条



较高体积分数设计结果



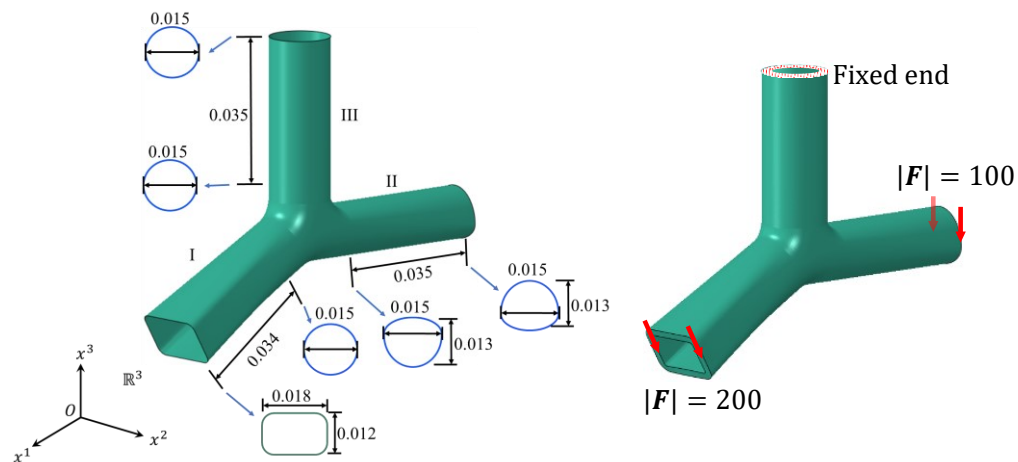
较低体积分数设计结果



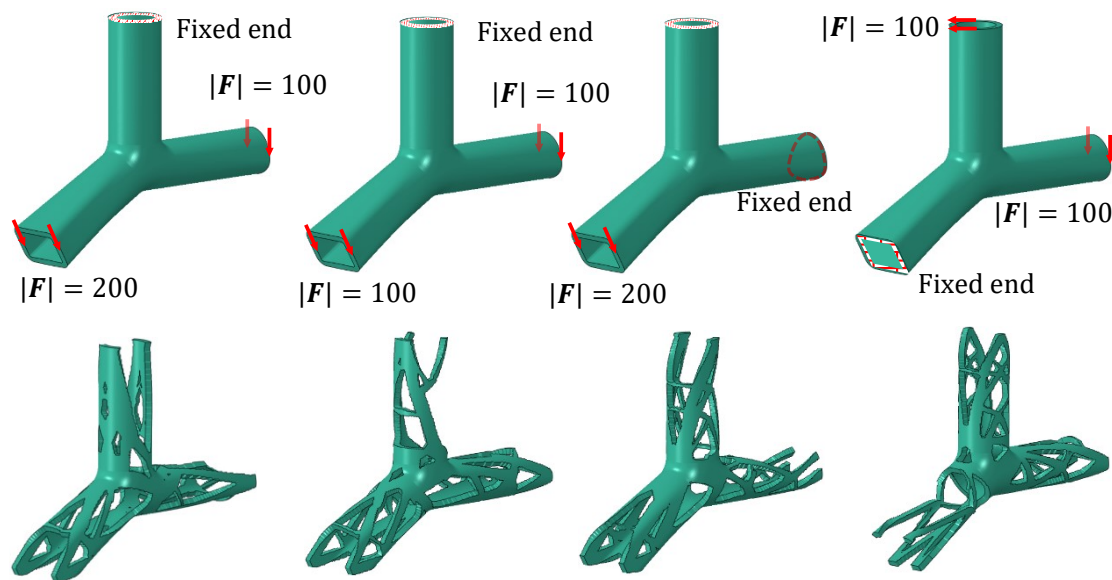
联系：加筋设计可以理解为蒙皮固定的拓扑设计

# 数值实验四：复杂工程算例的拓扑与夹芯层设计

## 问题设置

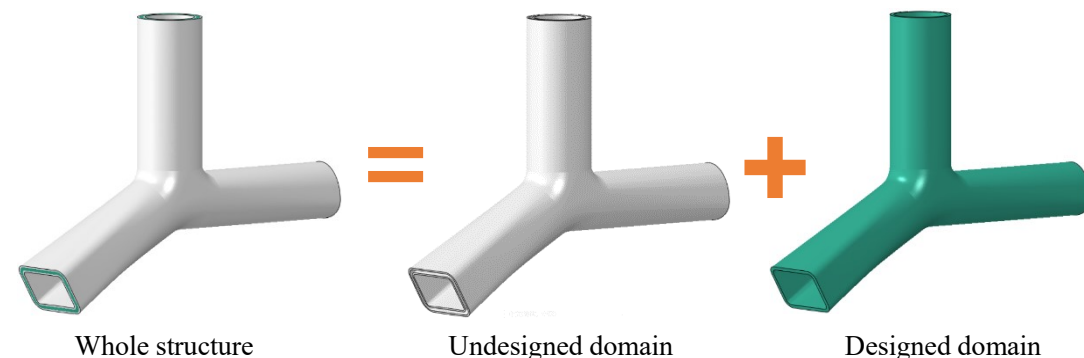


三通管道算例的几何尺寸与边界条件

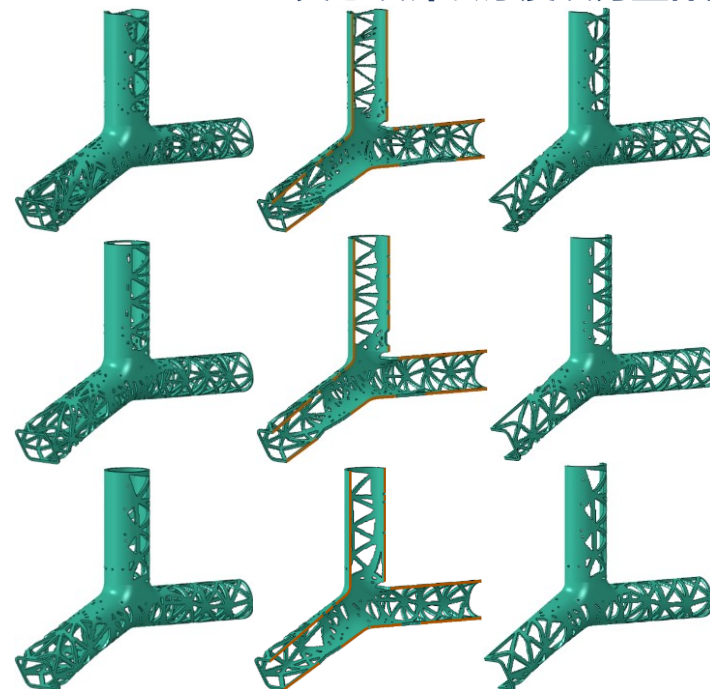


不同边界条件下的拓扑设计

## 夹芯结构设计



夹芯设计层厚度设为整体的一半



体积分数分别设置为 0.375, 0.45和0.55

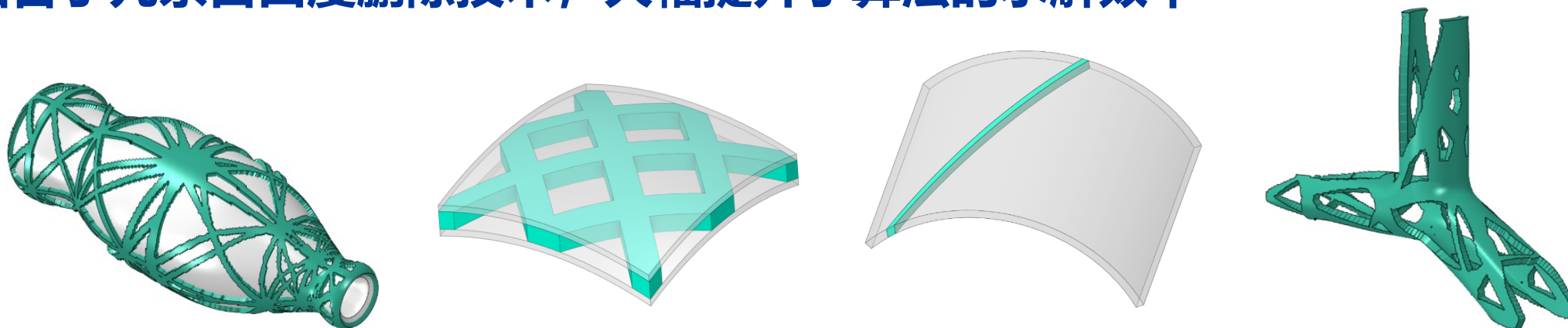
非设计层的杨氏模量 分别设置为0.15, 0.3和0.5

夹芯增强结构的最终设计

(多数为杆系和连续体结构，壳体模型难以实现建模与设计)

# 研究小结

- ✓ 基于实体模型和移动可变形组件法提出了一种复杂曲面薄壁结构优化的新方法
- ✓ 利用厚度坐标构造了嵌入式实体组件，实现了多类设计问题的统一建模和优化
- ✓ 结合了冗余自由度删除技术，大幅提升了算法的求解效率

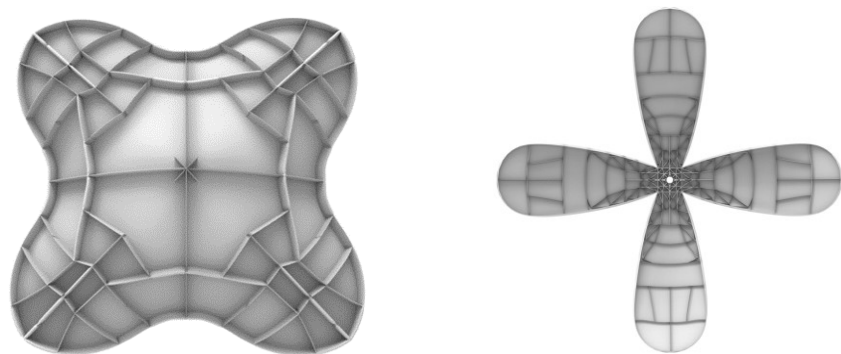


- 考虑加筋设计中筋条的截面优化，结合工程中常用的筋条制造方法
- 将复材本构融入夹芯板建模，考虑多材料、多杂化类型下的复合薄壁结构设计
- 采用高效的问题无关机器学习（PIML）分析框架，从而加速设计、提高精度

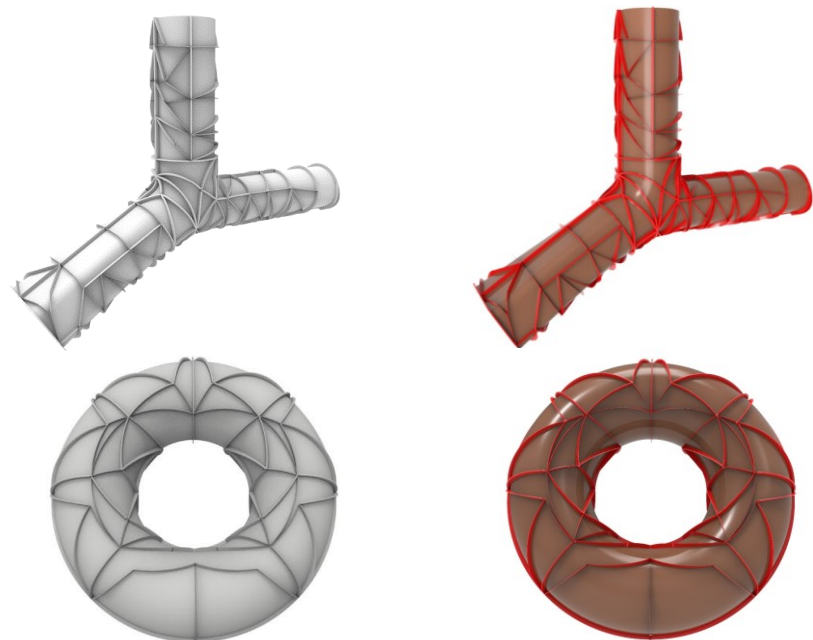
## 额外工作



## □ 复杂曲面薄壁结构的加筋设计



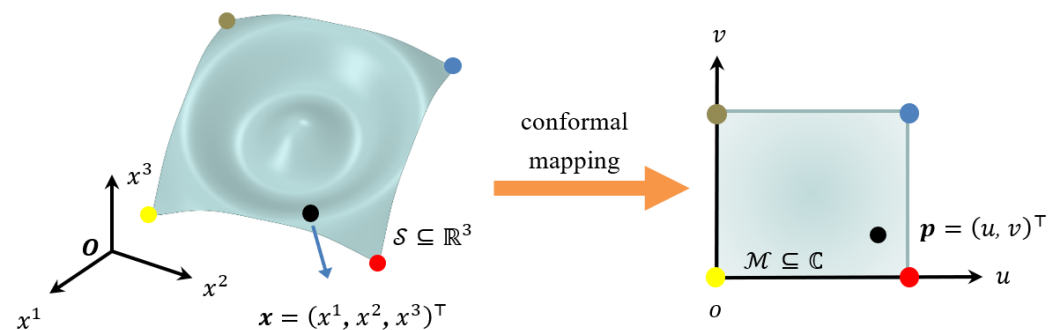
映射技术保证了加强筋与曲壳的连接性



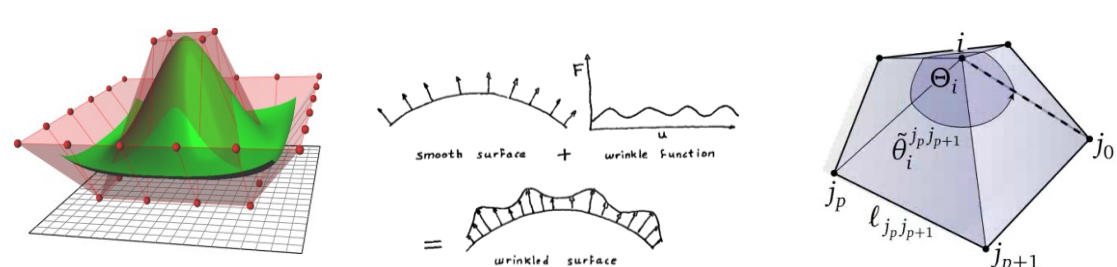
优化结果可以和CAD系统无缝连接

CMAME, 2023, 404: 115745.

## □ 保拓扑的薄壁结构形状设计



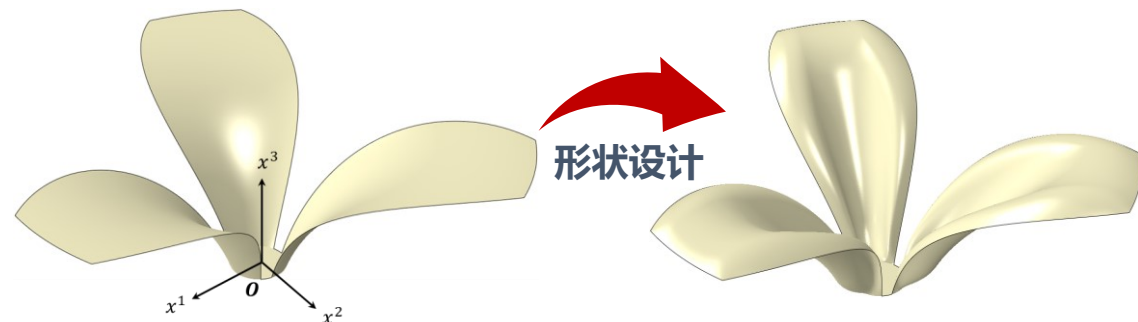
采用共形参数化技术对壳体结构进行降维



形状空间的构建

形状扰动形成全新构型

面向网格的离散法向

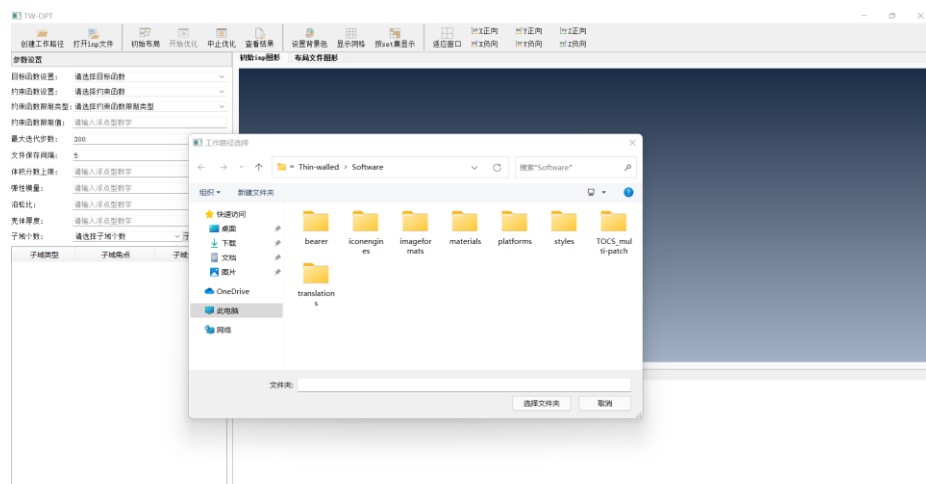


基于结构优化思想的花瓣构型解读

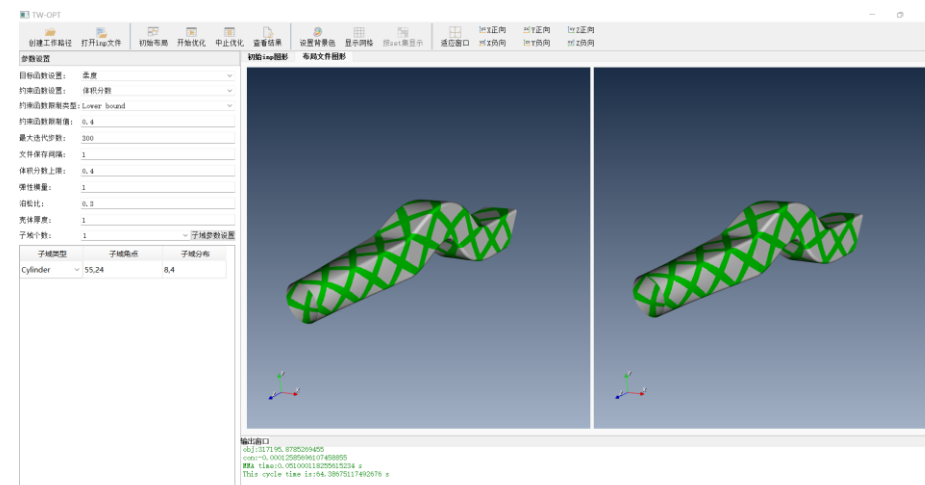
SMO, 2025, 68(10): 1-23.



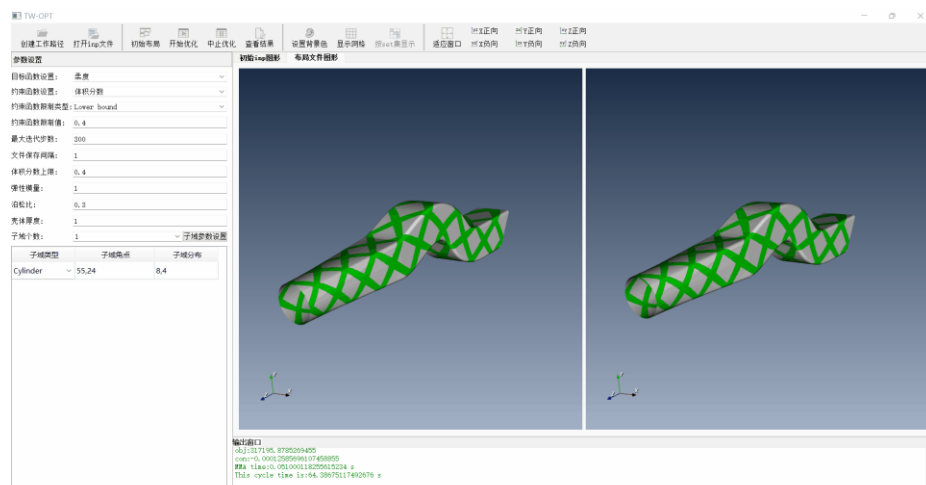
## 面向复杂曲面薄壁结构的专用化工业设计软件



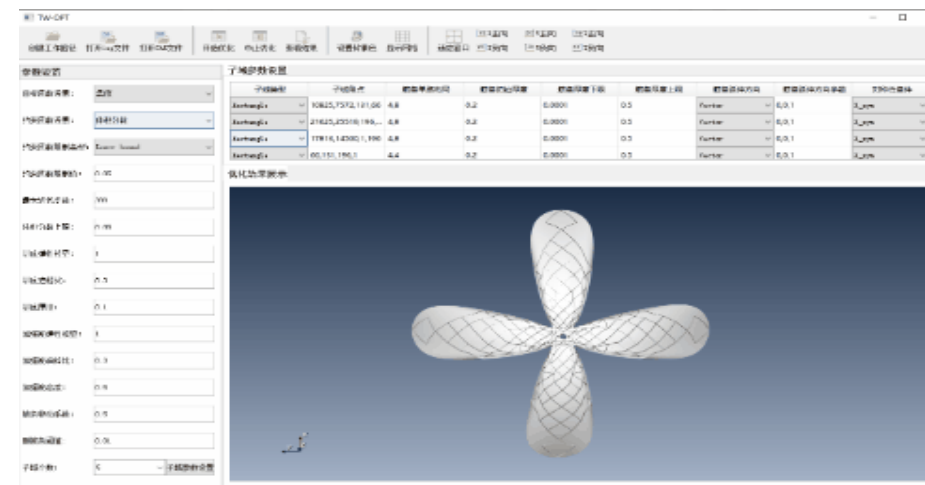
路径选择与文件设置



初始布局



拓扑优化模块



加筋设计模块

后续可先泛化为软件平台，再进一步集成多样化的工程解决方案

## 总结与展望

# 总结与展望

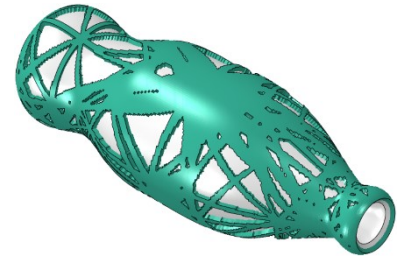
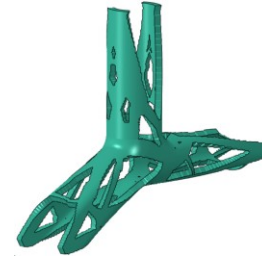
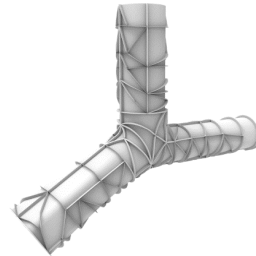
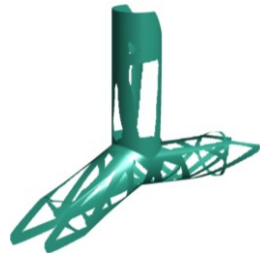
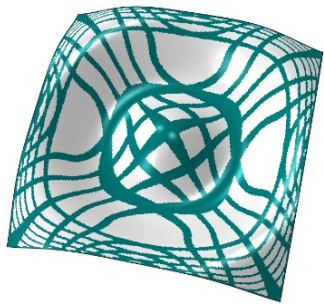
- ✓ 基于移动可变形组件法和共形参数化技术，构建了一种适用于壳体模型的显式拓扑优化算法
- ✓ 基于嵌入式形貌和样条组件描述，提出一种薄壁结构的显式形貌设计方法
- ✓ 发展了一种基于实体模型的薄壁结构显式设计方法，采用移动可变形组件法实现了拓扑优化、加筋优化与夹芯层优化问题的统一建模与求解
- 复杂问题/响应的推广：稳定性/基频/电磁/曲面流/软材料...
- 用结构设计算法来调控结构，例如控制裂纹扩展路径、控制曲面波导路径
- 结合先进制造技术，考虑制造工艺...
- 结合数据/人工智能...



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谢谢大家

恳请各位老师批评指正



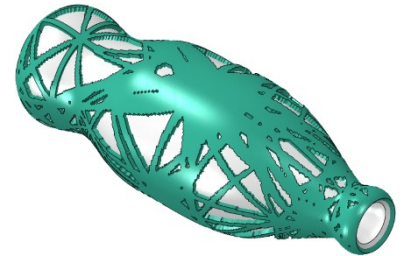
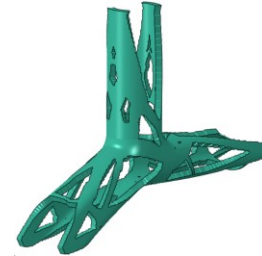
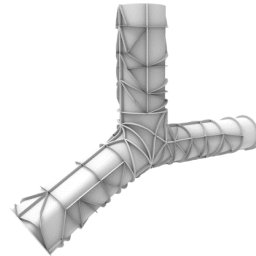
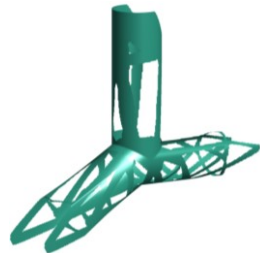
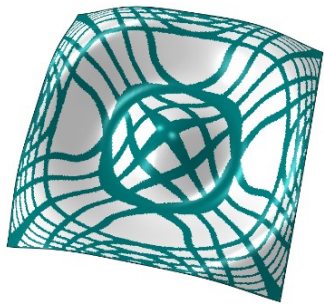
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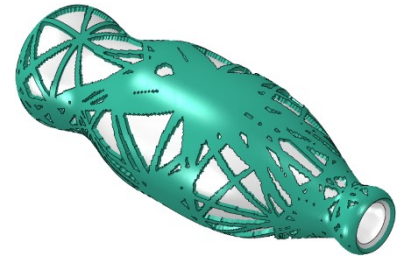
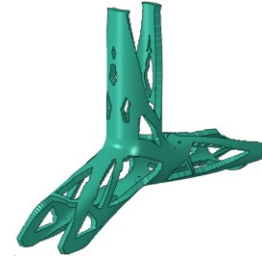
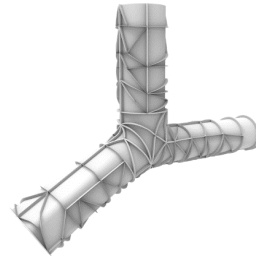
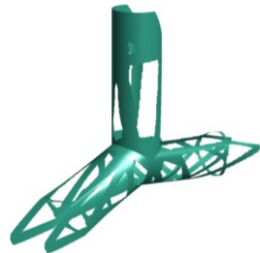
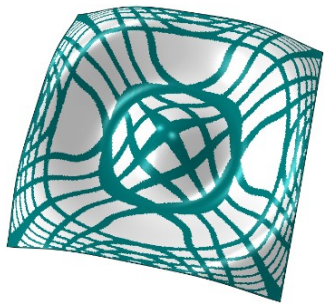




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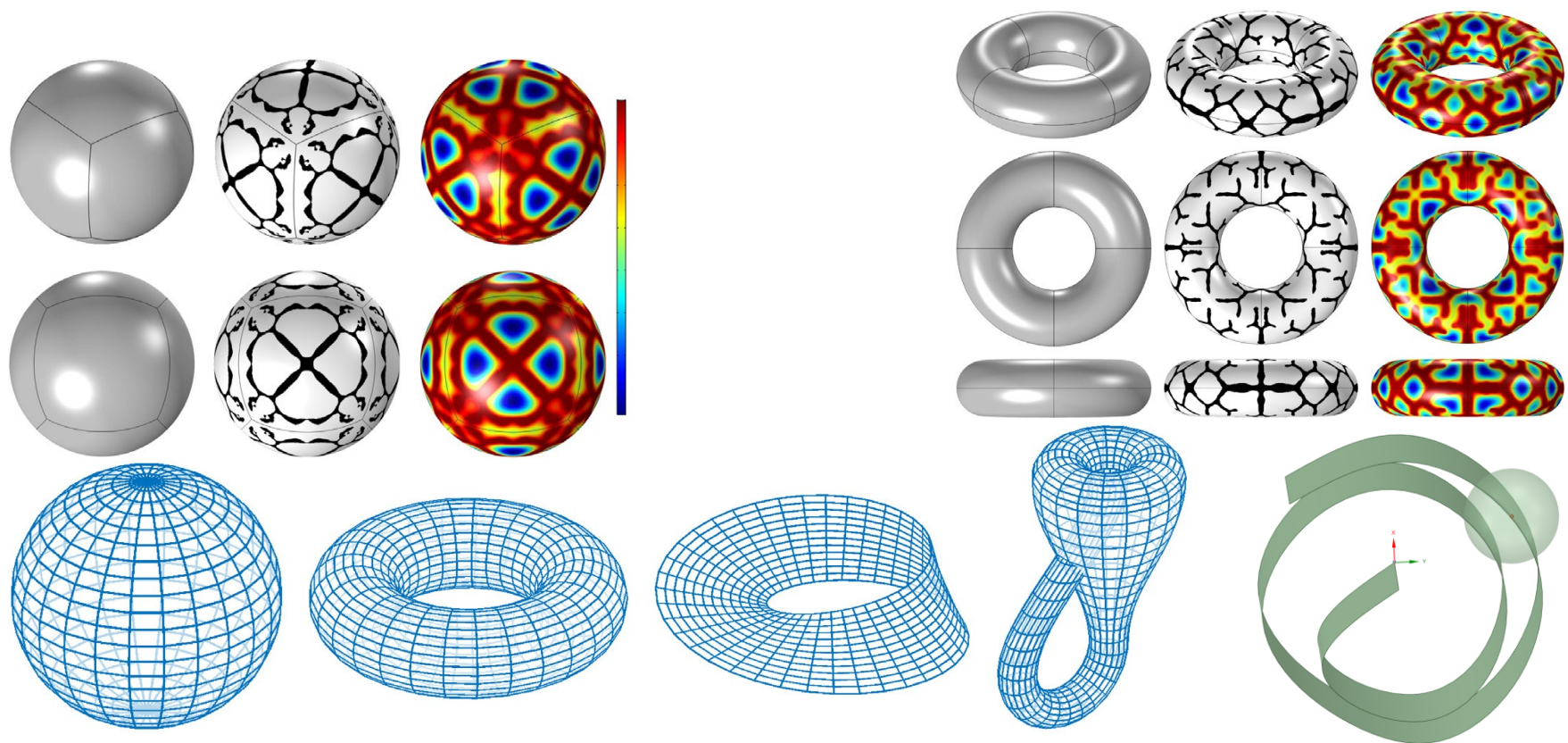
谢谢大家

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## □ 2-流形上的密度法拓扑优化 (Deng et. al., *CMAME* 364 (2020): 112937)



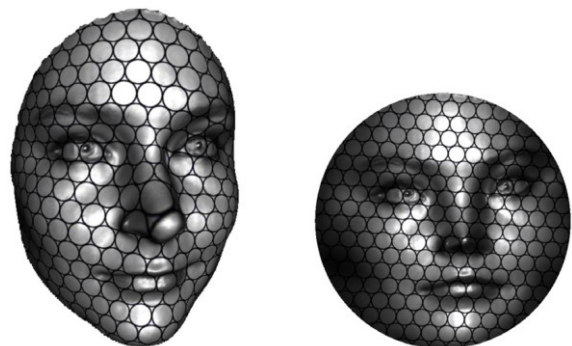
## □ 曲面上定义的过滤格式

$$\begin{cases} -r^2 \nabla^2 \tilde{\rho} + \tilde{\rho} = \rho, & \text{on } \Sigma_S \\ \frac{\partial \tilde{\rho}}{\partial \mathbf{n}} = 0, & \text{at } \partial \Sigma_S \end{cases}$$

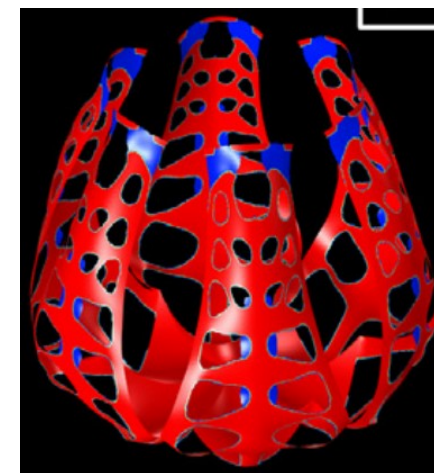
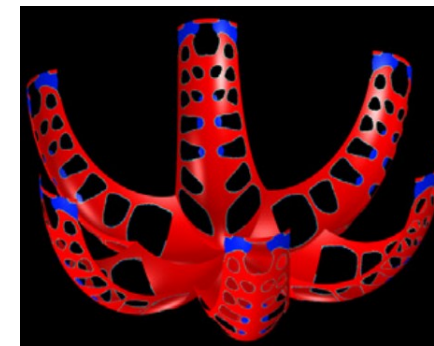
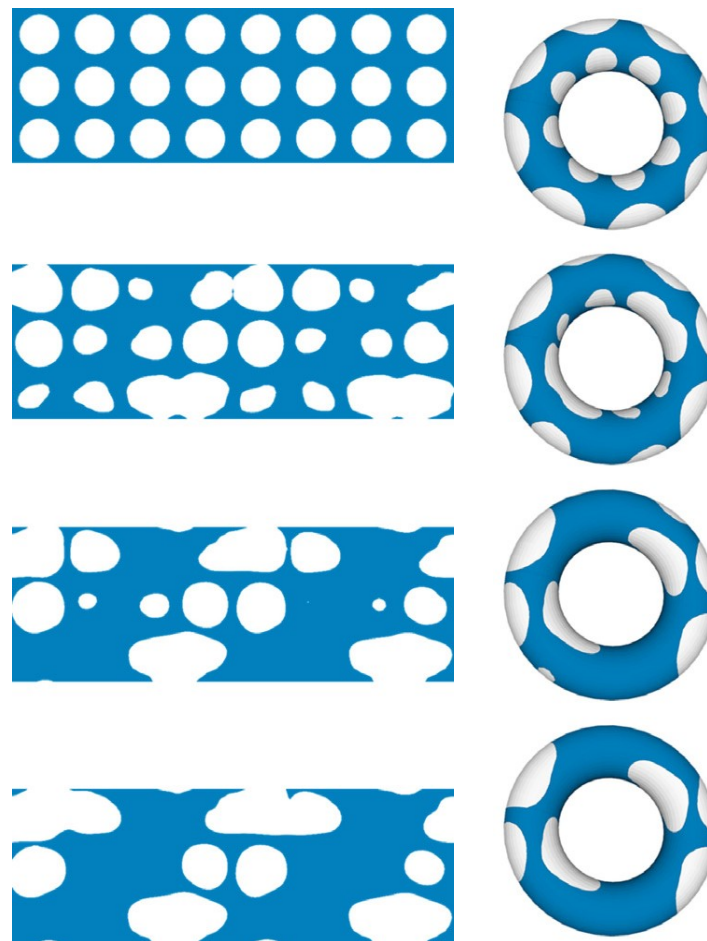
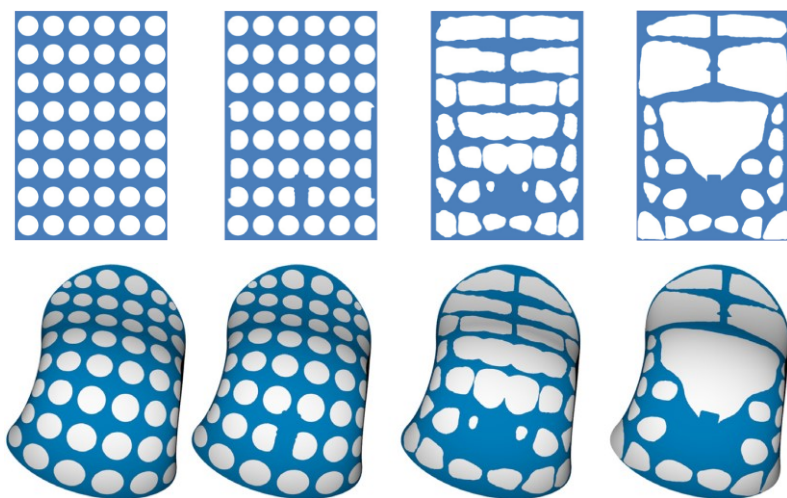


$$\begin{cases} \nabla_s \cdot (-r^2 \nabla_s \tilde{\gamma}) + \tilde{\gamma} = \gamma, & \text{on } \Sigma_S \\ -r^2 \nabla_s \tilde{\gamma} \cdot \boldsymbol{\tau} = 0, & \text{at } \partial \Sigma_S \end{cases}$$

## □ 表面上的水平集拓扑优化 (Ye et. al., *CMAME* 344 (2019): 164-185)



Conformal geometry theory



铁磁软物质结构设计

Conformal mapping + X-LSM >>> T.O. on Surfaces based on LSM method