

Full length article

Explicit topology optimization for fundamental frequency maximization of complex curved thin-walled stiffened structures

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ABSTRACT

To address the challenge of maximizing the fundamental frequency of complex thin-walled stiffened shells, this paper proposes an explicit topology optimization method based on the Moving Morphable Components (MMC) framework. Existing dynamic optimization approaches for stiffened structures are largely confined to flat plates or simple regular surfaces, and are difficult to extend to complex curved manifolds. To this end, a surface parametric mapping technique is utilized to establish a geometric correspondence between the 3D physical space and the 2D parametric domain. An adaptive ground structure method driven by shared nodes is then developed to update stiffener layouts while maintaining geometric conformity with the base surface. By directly employing the explicit geometric parameters of the stiffeners as design variables, the method dynamically updates the topological layout within the parametric domain, avoiding topological singularities induced by excessive component overlapping. Furthermore, to tackle artificial localized modes that may occur in fundamental frequency optimization, a stiffness-mass decoupled thickness-density double penalty mechanism is proposed. This mechanism penalizes redundant stiffeners located in low strain energy regions, thereby suppressing the premature excitation of low-energy local skin vibrations. A series of numerical examples involving complex curved shells—ranging from highly curved surfaces to multi-patch stitched cabin sections—demonstrate that the proposed method yields optimal stiffened configurations with distinct topological features, free of grayscale elements. The optimized designs provide clear explicit geometric parameters for stiffener layout, which can facilitate subsequent CAD/CAE modeling and engineering assessment. Under strict volume constraints, the proposed framework achieves a substantial improvement in the structural fundamental frequency, demonstrating its effectiveness as a numerical design tool for complex thin-walled stiffened shells.

1. Introduction

Thin-walled structures are extensively applied in high-end equipment design within the aerospace, marine, and launch vehicle sectors due to their high specific stiffness, load-bearing efficiency, and space utilization [1–4]. However, the thin-walled nature of these structures often leads to insufficient global stiffness and weak buckling resistance. It also leads to low fundamental natural frequencies. As a result, these structures are susceptible to resonance under dynamic excitations, such as wind loads and mechanical vibrations, during service. Such resonance may compromise structural integrity and fatigue life. Consequently,

introducing stiffeners to elevate the fundamental frequency has become a key design strategy to improve dynamic characteristics and avoid resonant failure. However, the optimal design of stiffeners remains challenging. This is because the stiffener cross-sectional geometry, topological layout, and base-skin curvature are strongly coupled. Therefore, determining an optimal stiffener configuration under complex geometric constraints is still a key issue in structural optimization.

Early research on stiffened structures primarily focused on size and shape optimization, mainly adjusting parameters like stiffener height, thickness, and spacing. For instance, Putra et al. [5] and Dong et al. [6] utilized hybrid genetic algorithms and bio-inspired growth mechanisms,

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respectively, to achieve lightweight stiffener layouts for flat plates or simple shells. Zhao et al. [7] and Zhang et al. [8] further explored the concurrent optimization of cross-sectional dimensions and layouts through hierarchical optimization strategies and load path visualization. While effective in specific applications, these methods are restricted by predefined parametric models, making it difficult to explore novel topological configurations. In contrast, topology optimization offers greater design freedom. It can naturally integrate size, shape, and layout optimization, and has gradually become the mainstream research paradigm in this field [9–16].

Within the framework of density-based implicit topology optimization, researchers have developed various stiffener design strategies. Ansola et al. [17] and Aage et al. [18] achieved refined stiffener microstructures using multi-layer solid elements or ultra-high-resolution voxel meshes, but at a prohibitive computational cost. To improve efficiency, the Smeared Stiffener Method (SSM) and its improved versions (NSSM) have been widely adopted [19–21]. Although these methods significantly reduce modeling complexity by homogenizing stiffener stiffness into the base skin, fundamentally, their optimization models are based on equivalent anisotropic plates rather than real discrete stiffened structures. The geometric discrepancy between this equivalence-based mechanical modeling and the actual physical configuration may fail to adequately capture the suppression effect of discrete stiffeners on localized modes, thereby introducing potential performance prediction errors when optimization results are converted into real engineering structures. Furthermore, while Liu et al. [22] and Wang et al. [23] attempted to generate distinct stiffeners through specific constraints or filters, their approaches still mainly target relatively regular geometries or require additional geometric handling when extended to complex curved surfaces.

To overcome the limitations of implicit methods regarding geometric description and mechanical modeling consistency, explicit topology optimization methods typified by the Moving Morphable Components (MMC) approach have emerged [24–27]. By transforming topology optimization into the parameter optimization of geometric components, this approach inherently offers fewer design variables, distinct boundaries, and easier CAD reconstruction. Most importantly, the MMC framework, based on a Lagrangian description, allows for the direct use of realistic discrete stiffener models for both mechanical analysis and optimization, ensuring strict consistency between the numerical optimization model and the final physical configuration. Jiang et al. [28] and Guo et al. [29] pioneered the application of MMC to the static load-bearing and impact-resistant design of flat shells. For more complex curved structures, Huo et al. [30–32] introduced the Computational Conformal Mapping (CCM) technique, establishing an explicit topology optimization framework for complex manifold surfaces. However, these MMC-based studies on complex surfaces have largely focused on static problems such as compliance minimization. The fundamental frequency maximization of complex curved stiffened shells remains insufficiently studied.

Maximizing the fundamental frequency of stiffened structures on complex curved surfaces presents significantly greater challenges than static problems. First, dynamic optimization is highly susceptible to mode switching and artificial localized modes, which trigger numerical instability during the optimization process. Second, existing research on frequency optimization is mostly confined to simple plates or cylindrical shells [33–35], or relies on heuristic algorithms [36–38], which cannot rigorously ensure geometric compatibility and gradient accuracy on complex manifolds. Consequently, efficiently computing dynamic sensitivities and suppressing numerical instability while maintaining high model fidelity remain critical bottlenecks in extending explicit topology optimization methods to practical engineering applications.

To address this gap, this paper proposes a novel explicit topology optimization method for maximizing the fundamental frequency of complex curved stiffened thin-walled structures. Under the MMC framework, the method first utilizes the CCM technique to establish a conformal mapping between a 3D manifold surface and a 2D parametric domain. This mapping enables stiffener layouts to be described and

optimized on irregular curved surfaces. Subsequently, a Shared-Node Driven Adaptive Ground Structure Method is introduced to dynamically update the stiffener layout in the parametric domain, fundamentally preventing topological singularities caused by component overlap. To suppress artificial localized modes associated with weakened stiffeners during frequency optimization, a dual density-thickness penalty mechanism is developed. The mechanism decouples the equivalent mass and bending stiffness of weak stiffeners and improves convergence robustness. General shell elements are adopted for finite element analysis. Analytical sensitivities are derived to enable gradient-based optimization. Because the stiffeners are described by explicit geometric parameters, the optimized results provide explicit geometric parameters to support subsequent CAD reconstruction after component screening, without density-field thresholding or boundary reconstruction. Numerical examples show that the method can generate stiffened configurations with clear boundaries and improved fundamental frequencies.

The remainder of this paper is organized as follows: Section 2 details the explicit geometric modeling framework for stiffened shells on complex surfaces based on CCM. Section 3 formulates the fundamental frequency maximization problem and derives the analytical sensitivities with respect to the explicit geometric parameters. Section 4 outlines the numerical implementation and solution strategies. Section 5 validates the proposed method's effectiveness and robustness through several representative numerical examples. Finally, Section 6 provides concluding remarks and outlines directions for future research.

2. Theoretical foundations

Designing stiffener layouts on complex free-form surfaces presents significant 3D geometric challenges. Performing topology optimization directly in curved physical space entails severe geometric compatibility issues, particularly in ensuring that stiffener components strictly adhere to the shell curvature during the optimization process without unphysical penetration or detachment. To address these issues, this section constructs a geometric modeling framework based on the MMC method and CCM technique. The core idea of this framework is to use the parametric domain to organize the reference positions of the stiffener skeleton, while generating the corresponding stiffener geometry on the physical curved surface through the established conformal mapping relationship. First, conformal mapping flattens the complex shell mid-surface into a 2D parametric domain, establishing a parameterized coordinate system with angle-preserving properties. Second, within this regular 2D parametric domain, the skeleton layout of the stiffener components is determined based on the Lagrangian description. Finally, utilizing the bijective mapping relationship, the skeleton geometric information is rapidly inversely mapped from the parametric domain onto the physical shell mid-surface. This is followed by normal extension and lateral thickening directly on the physical surface, thereby constructing a mechanical model for finite element analysis and optimization. This strategy ensures that the physical dimensions (thickness and height) of the stiffeners remain independent of mapping distortions, achieving concurrent optimization of topology and dimensions for stiffened structures on complex surfaces while maintaining geometric fidelity.

2.1. Explicit geometric construction method for stiffener components based on MMC

To achieve precise control over stiffened structures, this section first establishes a general explicit geometric modeling paradigm for stiffeners. Based on the MMC method, this paradigm adopts a hierarchical construction logic: first defining the skeleton via endpoints, then generating the mid-surface through normal extension, and finally constructing the solid volume via lateral thickening. Notably, while the position variables of the stiffeners are defined in the 2D parametric domain M , the actual generation of their solid geometry occurs in the 3D physical space to preserve physical realism.

2.1.1. Hierarchical Lagrangian description of stiffener components

In contrast to implicit description frameworks, this study employs a Lagrangian description [39] to explicitly represent structural geometry. Rather than relying on a fixed background mesh, this framework governs the layout and shape of components through a set of explicit design variables. The construction of a 3D stiffener solid can be decomposed into the following three hierarchical steps:

Definition of the Stiffener Skeleton. The geometric foundation of a stiffener is its central skeleton line. In the parametric domain $\mathcal{M}$, a local coordinate system (μ, η, ζ) is established. The skeleton $\overline{P_1 P_2}$ of a stiffener is uniquely determined by its two endpoint coordinates P_1 and P_2 . Any point $P(\mu)$ on this skeleton can be expressed via linear interpolation:

$$P(\mu) = (1 - \mu)P_1 + \mu P_2, \mu \in [0, 1], \quad (1)$$

where μ is the dimensionless local coordinate along the skeleton's length. This skeleton line is subsequently mapped onto the physical shell mid-surface S via inverse conformal mapping.

Generation of the Stiffener Mid-surface. In physical space, the mid-surface S_0 of the stiffener is generated by extending the mapped skeleton line along the normal to the shell surface. This step determines the height profile of the stiffener. Introducing the local coordinate $\eta \in [0, 1]$ along the height direction, the physical height parameter h , and the unit outward normal vector $\mathbf{n}$ of the physical shell mid-surface, any point on the stiffener mid-surface can be described as:

$$S_0(\mu, \eta) = P_{\text{phy}}(\mu) + \eta h \mathbf{n}, \quad (2)$$

where $P_{\text{phy}}(\mu)$ is the corresponding point of the skeleton on the physical surface. The rigorous inverse mapping mechanism from the parametric skeleton to the physical space is detailed in Sections 2.2 and 2.3.

Construction of the Stiffener Solid Volume. Finally, the 3D solid geometry of the stiffener is formed by laterally thickening the mid-surface. Let t be the physical thickness of the stiffener and $\zeta \in [-1/2, 1/2]$ be the local coordinate along the thickness direction. Let $\mathbf{n}_{\text{side}}$ denote the unit side normal vector, which is perpendicular to the skeleton and lies within the tangent plane in physical space. As shown in Fig. 1(b), the closed boundary $\partial\Omega$ of the stiffener solid is precisely enclosed by six surfaces, $\mathcal{S}_1$ through $\mathcal{S}_6$. The geometry of any point on these boundaries is explicitly defined as:

$$\begin{aligned} \mathcal{S}_1(\mu, \eta) &= S_0(\mu, \eta) + \frac{t}{2} \mathbf{n}_{\text{side}}, & \mathcal{S}_2(\mu, \eta) &= S_0(\mu, \eta) - \frac{t}{2} \mathbf{n}_{\text{side}}, \\ \mathcal{S}_3(\eta, \zeta) &= S_0(0, \eta) + \zeta t \mathbf{n}_{\text{side}}, & \mathcal{S}_4(\eta, \zeta) &= S_0(1, \eta) + \zeta t \mathbf{n}_{\text{side}}, \\ \mathcal{S}_5(\mu, \zeta) &= S_0(\mu, 0) + \zeta t \mathbf{n}_{\text{side}}, & \mathcal{S}_6(\mu, \zeta) &= S_0(\mu, 1) + \zeta t \mathbf{n}_{\text{side}}. \end{aligned} \quad (3)$$

Here, $\mathcal{S}_1$ and $\mathcal{S}_2$ represent the two main side surfaces along the length; $\mathcal{S}_3$ and $\mathcal{S}_4$ denote the start and end sections; and $\mathcal{S}_5$ and $\mathcal{S}_6$

correspond to the bottom (shell interface) and top surfaces, respectively. This set of equations fully defines the 3D stiffener component with physical thickness t and height h , ensuring a mathematically closed representation of the 3D stiffener geometry.

2.1.2. Shared-node driven adaptive ground structure method

During the optimization process, the unconstrained movement of independent discrete stiffeners often leads to disordered layouts and non-physical overlaps. To overcome this, we introduce the Shared-Node Driven Adaptive Ground Structure (SN-AGS) method to construct the initial layout and regularize the topological evolution [40]. As illustrated in Fig. 2(a), this method first discretizes the parametric domain into regular rectangular patches and arranges design nodes at the corners and centers of each rectangular patch. Based on this discretization, initial stiffeners are generated by connecting adjacent design nodes (e.g., connecting the center node to corner nodes). This procedure forms an initial ground structure with inherent connectivity. Under this framework, the geometric position of each stiffener is no longer defined by independent coordinate variables but is directly driven by its terminal design nodes. Because adjacent stiffeners naturally share these terminal nodes, any positional update of a design node inherently triggers a coordinated movement of all connected stiffeners. This maintains geometric continuity and continuous load transmission paths at the nodes.

Furthermore, to ensure structural boundary integrity and seamless stiffener transitions across multi-patch surfaces, differentiated kinematic constraints are applied to various types of design nodes. Specifically, nodes at the four vertices of the parametric domain remain stationary to anchor the global topology. Boundary nodes are permitted to slide only along the parametric domain boundaries to guarantee the presence of stiffeners at structural edges and facilitate smooth transitions with adjacent regions. Meanwhile, internal nodes are restricted to move strictly within a predefined local rectangular neighborhood around their initial positions. This prevents node crossover and stiffener geometric interpenetration, thereby significantly reducing the complexity of finite element modeling and analysis.

To facilitate topological changes (specifically, the reduction of stiffener numbers) within the explicit framework, a thickness-based "component deletion" mechanism is adopted. As shown in Fig. 2(b-c), upon convergence of the optimization, the system automatically identifies and removes "weakened stiffeners" whose thickness t falls below a preset threshold, subsequently reconstructing the topological connectivity based on the remaining valid components. This mechanism combines the MMC method's direct geometric control with the flexibility of traditional topology optimization. In fundamental frequency maximization, some isolated or "floating" stiffeners may appear after deletion. These components are not necessarily topological defects. From the perspective of dynamic response, they can act as local mass/stiffness

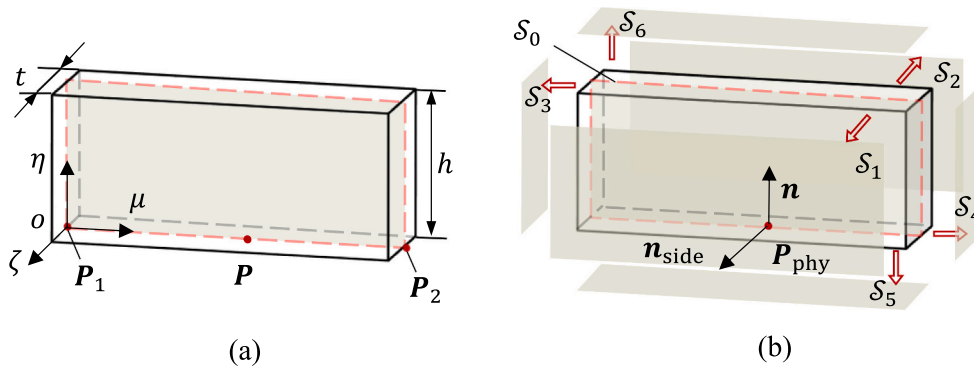


Fig. 1. Hierarchical geometric description of a stiffener component: (a) Skeleton definition in the parametric domain $\mathcal{M}$; (b) Hexahedral solid generated via normal mid-surface extension and lateral thickening in physical space.

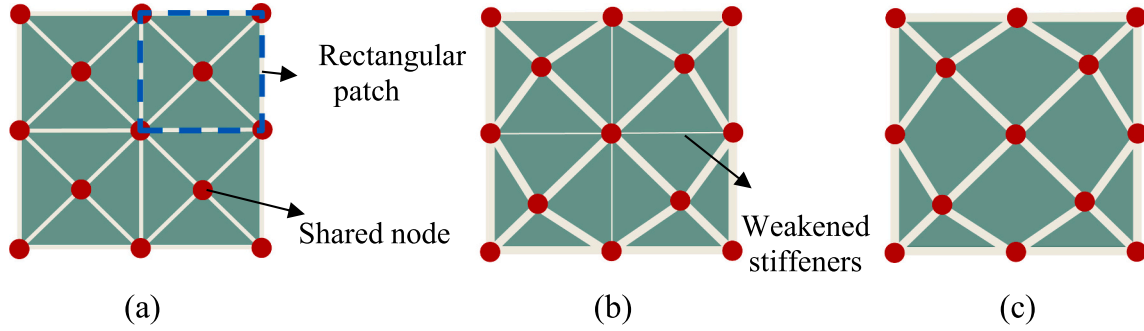


Fig. 2. Shared-Node Driven Adaptive Ground Structure Method: (a) Design nodes and initial ground structure; (b) Identification of weakened stiffeners after optimization; (c) Final explicit topology after component deletion.

tuners. They help suppress artificial localized modes in vulnerable regions. Therefore, the final optimized design can be exported as explicit stiffener parameters without image processing or geometric reconstruction.

2.2. Conformal mapping parameterization of complex surfaces

The explicit geometric construction paradigm described previously relies on a flat parameter space and cannot be directly applied to general curved manifolds. To address this, we introduce the CCM technique proposed in [41,42] to establish a bijective mapping between the physical shell mid-surface $\mathcal{S}$ and a 2D parametric domain $\mathcal{M}$. Rather than solving high-dimensional nonlinear equations directly, this study formulates the complex manifold parameterization as a tractable system of linear elliptic equations. This is achieved via a two-step strategy that constructs the final conformal mapping by composing two distinct mappings governed by specific Beltrami coefficients.

This parameterization process begins with the construction of a topological homeomorphism via a harmonic map. Specifically, the simply

connected open surface $\mathcal{S}$ is mapped to the unit disk $\mathcal{D}$ on the complex plane (Fig. 3(a)). The mapping $f_1 : \mathcal{S} \rightarrow \mathcal{D}$ is constructed to satisfy the following Laplace boundary value problem:

$$\begin{cases} \Delta_{\mathcal{S}} f_1 = 0, & \text{on } \mathcal{S}, \\ f_1(\partial \mathcal{S}) = \partial \mathcal{D}, \end{cases} \quad (4)$$

where $\Delta_{\mathcal{S}}$ is the Laplace–Beltrami operator on the surface. This equation can be efficiently solved using the Finite Element Method [43]. However, while f_1 achieves planar parameterization, it is generally not conformal due to the lack of control over angular distortion, which can induce severe geometric warping. To correct this, a second quasi-conformal mapping $f_2 : \mathcal{D} \rightarrow \mathcal{M}$ is introduced to rectify the parametric domain geometry and eliminate angular distortions (Fig. 3(b)). Let the Beltrami coefficient of the inverse mapping f_1^{-1} be $\mu_{f_1^{-1}} = \rho + i\tau$. To counteract the angular deformation induced by f_1 , the mapping $f_2(x^1 + ix^2) = u(x^1, x^2) + iv(x^1, x^2)$ must satisfy the following generalized Laplace equations [41]:

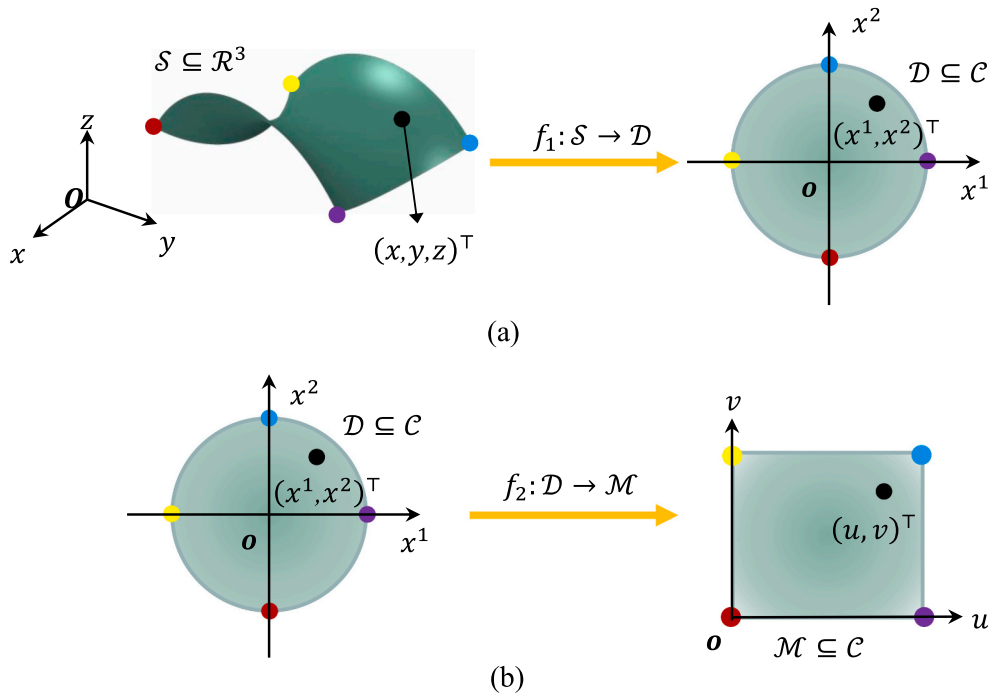


Fig. 3. Surface parameterization process based on composite quasi-conformal mapping: (a) Initial harmonic mapping f_1 to the unit disk $\mathcal{D}$ (exhibiting conformal distortion); (b) Quasi-conformal correction mapping f_2 and the final angle-preserving parametric domain $\mathcal{M}$.

$$\begin{cases} \nabla \cdot (\mathbf{A} \nabla u) = 0, \\ \nabla \cdot (\mathbf{A} \nabla v) = 0, \end{cases} \quad (5)$$

where $\nabla(\cdot) = (\partial(\cdot)/\partial x^1, \partial(\cdot)/\partial x^2)^\top$, and the coefficient matrix $\mathbf{A} =$

$\begin{pmatrix} \alpha_1 & \alpha_2 \\ \alpha_2 & \alpha_3 \end{pmatrix}$ encodes the geometric correction information. The coefficients are defined as:

$$\alpha_1 = \frac{(\rho - 1)^2 + \tau^2}{1 - \rho^2 - \tau^2}, \alpha_2 = -\frac{2\tau}{1 - \rho^2 - \tau^2}, \alpha_3 = \frac{(\rho + 1)^2 + \tau^2}{1 - \rho^2 - \tau^2}. \quad (6)$$

The resulting global mapping $f = f_2 \circ f_1 : \mathcal{S} \rightarrow \mathcal{M}$ possesses strict angle-preserving properties. This allows an orthogonal grid defined in the parametric domain $\mathcal{M}$ to determine the corresponding reference positions of stiffener nodes on the physical surface through the conformal mapping relation, thereby helping control geometric distortion of the stiffeners.

2.3. Inverse mapping-based modeling of stiffeners on complex surfaces

As illustrated in Fig. 4, a realistic thin-walled stiffened structure comprises a base skin and attached three-dimensional stiffeners. Because the stiffener components are initially defined in the 2D parametric domain (Section 2.1), constructing their 3D physical configuration on a curved substrate (Fig. 5) requires a precise inverse geometric reconstruction based on the conformal mapping established in Section 2.2. This section derives the surface normal vectors and stiffener tangent vectors in physical space through coordinate transformations rooted in differential geometry, thereby completing the inverse mapping modeling from 2D parametric components to 3D stiffeners on complex surfaces.

Specifically, the generation of the physical entity commences with the inverse mapping of the stiffener skeleton and subsequent normal extension (Fig. 6(a)(b)). Let a point $\mathbf{P} = (u, v)^\top$ in the parametric domain $\mathcal{M}$ lie on a stiffener skeleton; its corresponding point on the physical base surface $\mathbf{P}' = f^{-1}(\mathbf{P})$ is obtained directly via inverse mapping. To construct the mid-surface, we calculate the unit outward normal vector $\mathbf{n}(\mathbf{P}')$ of the physical surface at point $\mathbf{P}'$, which is derived from the Jacobian matrix of the inverse mapping:

$$\mathbf{n}(\mathbf{P}') = \frac{\frac{\partial f^{-1}}{\partial u} \times \frac{\partial f^{-1}}{\partial v}}{\left\| \frac{\partial f^{-1}}{\partial u} \times \frac{\partial f^{-1}}{\partial v} \right\|}. \quad (7)$$

Substituting this normal vector into Eq. (2), the parametric equation of the stiffener mid-surface in physical space is determined as $\mathbf{S}_0^{\text{phy}}(u, v, \eta) = f^{-1}(u, v) + \eta \mathbf{n}(f^{-1}(u, v))$. This step accomplishes the first stage of inverse localization, mapping the mid-surface skeleton onto the physical curved surface.

Following the determination of the physical mid-surface, the side normal vector $\mathbf{n}_{\text{side}}$ in physical space must be determined to construct a solid with physical thickness t . Let the skeleton direction vector in the

parametric domain be $\mathbf{d} = (u_2 - u_1, v_2 - v_1)^\top$. Applying the pushforward operator in differential geometry, this vector is transformed by the Jacobian matrix of the inverse mapping. Upon normalization, the unit tangent vector $\boldsymbol{\tau}_P$ for the physical entity is obtained as:

$$\boldsymbol{\tau}_P = \frac{(u_2 - u_1) \frac{\partial f^{-1}}{\partial u} + (v_2 - v_1) \frac{\partial f^{-1}}{\partial v}}{\left\| (u_2 - u_1) \frac{\partial f^{-1}}{\partial u} + (v_2 - v_1) \frac{\partial f^{-1}}{\partial v} \right\|}. \quad (8)$$

Then, the unit side vector $\mathbf{n}_{\text{side}}$, perpendicular to the skeleton and tangent to the surface, is defined by the cross product:

$$\mathbf{n}_{\text{side}} = \frac{\boldsymbol{\tau}_P \times \mathbf{n}(\mathbf{P}')}{\|\boldsymbol{\tau}_P \times \mathbf{n}(\mathbf{P}')\|}. \quad (9)$$

At this point, all geometric elements required to construct the solid in physical space are obtained. Substituting the calculated $\mathbf{n}_{\text{side}}$ into Eq. (3), the boundary surfaces of the stiffener (e.g., $\mathbf{S}_1, \mathbf{S}_2$) can be precisely generated, as shown in Fig. 6(c).

Finally, to handle high-genus complex shell structures, this study adopts a “partition-map-stitch” strategy [30]. For instance, the cylindrical shell depicted in Fig. 7(a) can be transformed into a simply connected domain by cutting along a generatrix; meanwhile, the more complex composite shell shown in Fig. 7(b) can be segmented into multiple sub-regions based on its geometric features, with a conformal mapping established for each patch independently. Enforced by the boundary node constraints defined in Section 2.1.2, stiffeners generated within each sub-region can align at the stitching interfaces and be assembled into a complete stiffened shell model. Based on these derivations, the geometric configuration of the stiffeners in physical space is explicitly driven by the position variables in the parametric domain and the physical dimension variables. This establishes a rigorous geometric foundation for the subsequent finite element analysis and topology optimization.

3. Problem statement and sensitivity analysis

3.1. Optimization formulation

This section formulates a topology optimization model to maximize the fundamental frequency of thin-walled stiffened curved shell structures, based on their exact physical configuration. Let the physical domain occupied by the entire stiffened shell be denoted as $\mathcal{B} = \bigcup_{i=0}^{n_c} \mathcal{B}_i$, where $\mathcal{B}_0$ represents the fixed base skin region, $\mathcal{B}_i$ denotes the region occupied by the i -th stiffener to be designed, and n_c is the total number of stiffeners. The design variables comprise the coordinates of the skeleton driving nodes for all stiffeners in the parametric domain, denoted as $\mathbf{D}_p = (\mathbf{P}_1, \dots, \mathbf{P}_{n_n})^\top$ (n_n being the total number of driving nodes), and the thickness parameters of the stiffeners, denoted as $\mathbf{D}_t = (t_1, \dots, t_{n_c})^\top$. Thus, the global design variable vector is defined as $\mathbf{D} = (\mathbf{D}_p^\top, \mathbf{D}_t^\top)^\top$.

To account for the mechanical behavior of the complex curved structures spanning thin to moderately thick regimes, the structural

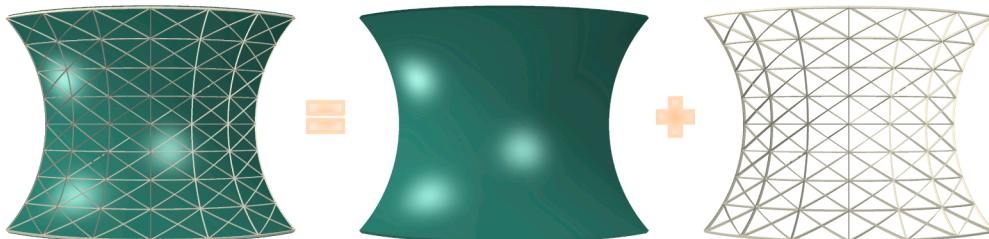


Fig. 4. Geometric composition of a free-form stiffened shell assembly of the base skin and 3D stiffener components.

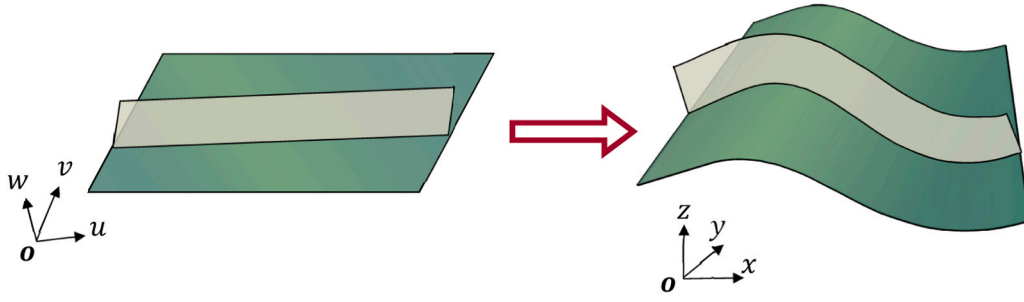


Fig. 5. Schematic of the 3D stiffener solid model on the physical surface.

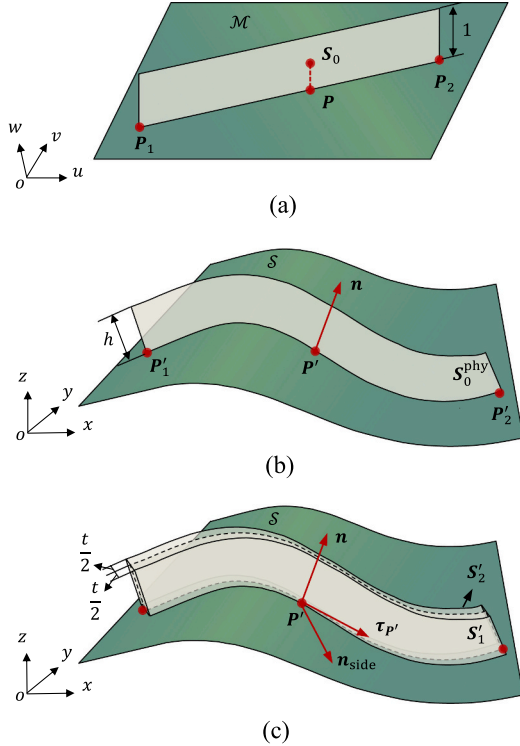


Fig. 6. Detailed inverse modeling workflow of stiffeners on complex surfaces: (a) Skeleton in parametric domain; (b) Inverse mapping to physical surface and normal extension (along vector $\mathbf{n}$) to generate mid-surface S_0 ; (c) Lateral thickening (vector $\mathbf{n}_{\text{side}}$) to generate closed solid boundaries S'_1, S'_2 .

analysis is founded on general shell theory. Subject to a constraint on the upper bound of stiffener volume, the mathematical model seeking the maximization of the structure's first natural frequency can be formulated as follows:

$$\begin{aligned}
 &\text{Find :} && \mathbf{D} = (\mathbf{D}_p^\top, \mathbf{D}_t^\top)^\top, \mathbf{U}(\mathbf{D}) = (\mathbf{U}_0^\top, \dots, \mathbf{U}_{n_c}^\top)^\top, \lambda(\mathbf{D}) \\
 &\text{Minimize :} && -\lambda(\mathbf{D}) \\
 &\text{Subject to :} && \sum_{i=0}^{n_c} \int_{B_i} \boldsymbol{\varepsilon}(\mathbf{U}_i) : \mathbb{E} : \boldsymbol{\varepsilon}(\mathbf{V}_i) dV = \lambda(\mathbf{D}) \sum_{i=0}^{n_c} \int_{B_i} \rho \mathbf{U}_i \cdot \mathbf{V}_i dV, \forall \mathbf{V} \in \mathcal{H} \\
 &&& \text{ad}, \int_B \rho \mathbf{U} \cdot \mathbf{U} dV = 1, V(\mathbf{D}) = \sum_{i=1}^{n_c} \int_{B_i} dV \leq \bar{V} \int_{B_0} dV, \mathbf{D} \in \mathcal{H}_D. \quad (10)
 \end{aligned}$$

Here, $\lambda(\mathbf{D})$ denotes the first eigenvalue of the structure, which relates to the fundamental frequency f via $f(\mathbf{D}) = \sqrt{\lambda(\mathbf{D})}/(2\pi)$. Therefore, maximizing the fundamental frequency is equivalent to minimizing $-\lambda(\mathbf{D})$. $\mathbf{U}_i$ and $\mathbf{V}_i$ represent the actual and arbitrary virtual displacement fields in the i -th subdomain, respectively, both defined within the kinematically admissible space $\mathcal{H}_{\text{ad}}$. The symbol $\mathbb{E}$ is the fourth-order

constitutive tensor for isotropic linear elastic materials, $\boldsymbol{\varepsilon}(\cdot)$ is the strain tensor derived from general shell theory, and ρ is the material density. The third constraint equation in Eq. (10) imposes the global mass normalization condition to ensure the uniqueness of the eigenmode [44]. Finally, $\bar{V}$ specifies the upper limit on the stiffener volume fraction relative to the base skin, and $\mathcal{H}_D$ denotes the feasible domain of the design variables.

3.2. Sensitivity analysis

The proposed MMC-based stiffening optimization method for complex curved surfaces is inherently a boundary evolution problem within a Lagrangian framework. Consequently, shape sensitivity analysis [45, 46], coupled with the continuum mechanics concept of the material derivative, is employed to derive the analytical gradients of both the objective function and constraint functions with respect to the explicit geometric parameters.

First, without loss of generality, the weak form governing equation (variational formulation) for the first-order undamped free vibration of the structural system can be expressed as:

$$\int_{\mathcal{B}} \boldsymbol{\varepsilon}(\mathbf{U}) : \mathbb{E} : \boldsymbol{\varepsilon}(\mathbf{V}) dV = \lambda \int_{\mathcal{B}} \rho \mathbf{U} \cdot \mathbf{V} dV, \quad \forall \mathbf{V} \in \mathcal{H}_{\text{ad}}. \quad (11)$$

By introducing a fictitious time parameter τ to characterize the geometric evolution process of the global structural physical boundary ∂B driven by the design variables in the parameter domain, we take the material derivative of both sides of Eq. (11) with respect to τ . Applying the Reynolds Transport Theorem and accounting for the conversion between volume and boundary integrals, the expanded form is obtained as follows:

$$\begin{aligned}
 &\int_{\mathcal{B}} \boldsymbol{\varepsilon}(\mathbf{U}') : \mathbb{E} : \boldsymbol{\varepsilon}(\mathbf{V}) dV + \int_{\mathcal{B}} \boldsymbol{\varepsilon}(\mathbf{U}) : \mathbb{E} : \boldsymbol{\varepsilon}(\mathbf{V}') dV + \int_{\partial \mathcal{B}} (\boldsymbol{\varepsilon}(\mathbf{U}) : \mathbb{E} : \boldsymbol{\varepsilon}(\mathbf{V})) V_n dS \\
 &= \lambda' \int_{\mathcal{B}} \rho \mathbf{U} \cdot \mathbf{V} dV + \lambda \int_{\mathcal{B}} \rho \mathbf{U}' \cdot \mathbf{V} dV + \lambda \int_{\mathcal{B}} \rho \mathbf{U} \cdot \mathbf{V}' dV + \lambda \int_{\partial \mathcal{B}} \rho \mathbf{U} \cdot \mathbf{V} V_n dS. \quad (12)
 \end{aligned}$$

In the above equation, $(\cdot)'$ denotes the material derivative with respect to τ , and V_n represents the normal velocity of the boundary ∂B along the unit outward normal $\mathbf{n}_{\text{side}}$.

Given that the linear elastic structural dynamic system possesses self-adjoint properties, when the trial displacement field is chosen as the actual eigenmode (i.e., $\mathbf{U} = \mathbf{V}$), the material derivative of the displacement $\mathbf{U}$ can itself be regarded as an admissible virtual displacement field ($\mathbf{U}' \in \mathcal{H}_{\text{ad}}$). Consequently, substituting $\mathbf{U}'$ into Eq. (11) yields:

$$\int_{\mathcal{B}} \boldsymbol{\varepsilon}(\mathbf{U}') : \mathbb{E} : \boldsymbol{\varepsilon}(\mathbf{V}) dV = \lambda \int_{\mathcal{B}} \rho \mathbf{U}' \cdot \mathbf{V} dV. \quad (13)$$

Similarly, since the virtual displacement $\mathbf{V}$ is independent of the boundary shape evolution (i.e., $\mathbf{V}' = 0$ when $\mathbf{V} = \mathbf{U}$), the relevant terms can be eliminated. By substituting Eq. (13) into Eq. (12) and canceling out the identical volume integral terms, the global derivative of the

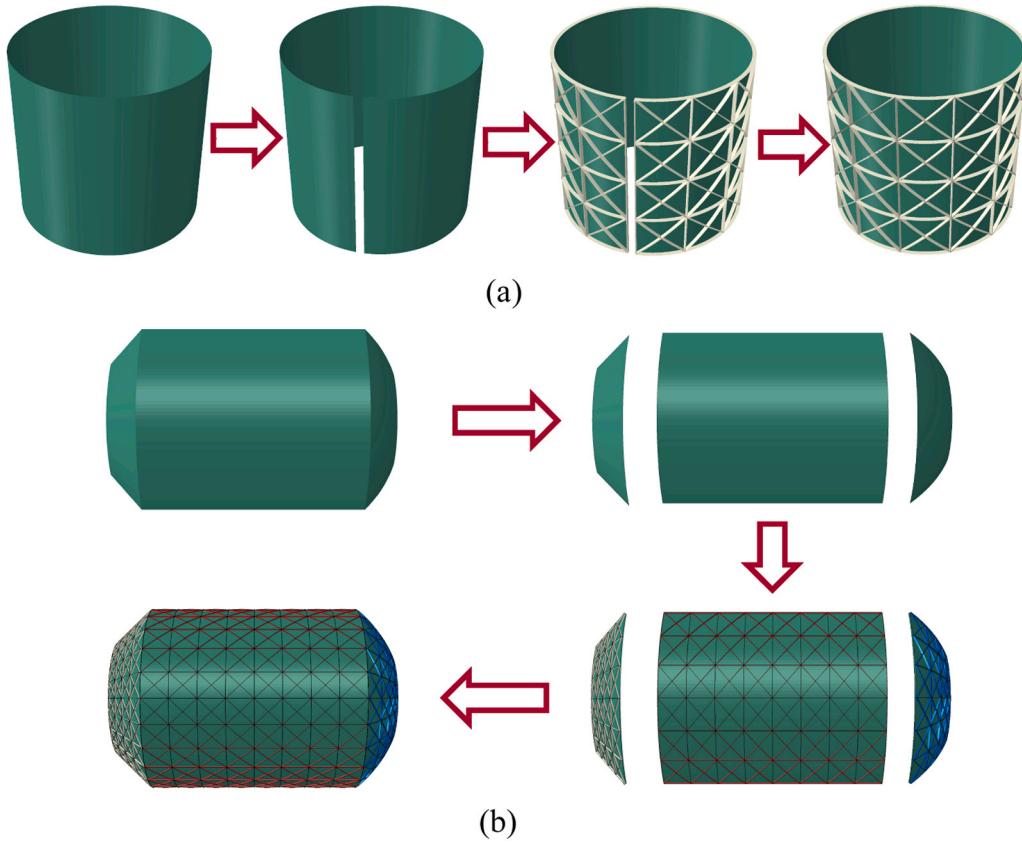


Fig. 7. Parameterization strategy for complex surface shells: (a) Cut-and-unfold for cylindrical structures; (b) Multi-patch segmentation and stitching for complex composite surfaces.

eigenvalue λ is finally derived as:

$$\lambda' = \frac{\int_{\partial\mathcal{B}} (\boldsymbol{\varepsilon}(\mathbf{U}) : \mathbb{E} : \boldsymbol{\varepsilon}(\mathbf{U}) - \lambda \rho \mathbf{U} \cdot \mathbf{U}) V_n dS}{\int_{\mathcal{B}} \rho \mathbf{U} \cdot \mathbf{U} dV}. \quad (14)$$

Since the mass-normalization condition (i.e., $\int_{\mathcal{B}} \rho \mathbf{U} \cdot \mathbf{U} dV = 1$) was introduced in Section 3.1, and considering that the boundary of the base skin $\mathcal{B}_0$ is fixed while boundary evolution occurs exclusively on the outer boundaries of the individual stiffeners, Eq. (14) can be significantly simplified and decomposed into the boundary integral contributions of each discrete stiffener:

$$\lambda' = \sum_{i=1}^{n_c} \int_{\partial\mathcal{B}_i \cap \partial\mathcal{B}} (\boldsymbol{\varepsilon}(\mathbf{U}) : \mathbb{E} : \boldsymbol{\varepsilon}(\mathbf{U}) - \lambda \rho \mathbf{U} \cdot \mathbf{U}) V_n^i dS. \quad (15)$$

Although Eq. (15) provides the classical continuum shape derivative, in the proposed framework, the normal velocity field V_n^i is unknown and cannot be directly linked to the design variables. It is imperative to establish an analytical chain between the physical boundary evolution and the explicit design variables $\mathbf{D} = (\mathbf{D}_p^\top, \mathbf{D}_t^\top)^\top$ in the two-dimensional parameter domain via differential geometric mapping.

For the i -th three-dimensional curved stiffener depicted in Fig. 6, considering thin-walled shell characteristics, its primary outer boundary is composed of the two lateral surfaces $\mathcal{S}_1^i$ and $\mathcal{S}_2^i$ (the end faces are negligible and thus omitted). The normal velocity field can be expressed as:

$$V_n^i = \begin{cases} \delta \mathcal{S}_1^i \cdot \mathbf{n}_{\text{side}}, & \text{for } \mathcal{S}_1^i, \\ \delta \mathcal{S}_2^i \cdot (-\mathbf{n}_{\text{side}}), & \text{for } \mathcal{S}_2^i. \end{cases} \quad (16)$$

According to the solid generation logic of Eq. (3), the lateral boundary surfaces are generated by thickening the mid-surface $\mathcal{S}_0^i$ along the lateral normal direction $\mathbf{n}_{\text{side}}$. Invoking the orthogonality property

between the variation of the normal vector and the normal vector itself ($\delta \mathbf{n}_{\text{side}} \cdot \mathbf{n}_{\text{side}} = 0$), Eq. (16) can be precisely reduced to a function depending solely on the variation of the mid-surface position $\delta \mathcal{S}_0^i$ and the thickness variation δt_i :

$$V_n^i = \begin{cases} \delta \mathcal{S}_0^i \cdot \mathbf{n}_{\text{side}} + \frac{1}{2} \delta t_i, & \text{on } \mathcal{S}_1^i, \\ -\delta \mathcal{S}_0^i \cdot \mathbf{n}_{\text{side}} + \frac{1}{2} \delta t_i, & \text{on } \mathcal{S}_2^i. \end{cases} \quad (17)$$

Furthermore, the infinitesimal variation of the mid-surface position $\delta \mathcal{S}_0^i$ on the physical curved surface is entirely determined by the perturbations of its corresponding coordinates (u, v) in the parameter domain. By combining the conformal inverse mapping equation $f^{-1}(u, v)$ with Eq. (2), and employing the chain rule along with the push-forward Jacobian matrix, we obtain:

$$\begin{aligned} \delta \mathcal{S}_0^i &= \frac{\partial \mathcal{S}_0^i}{\partial u} \delta u + \frac{\partial \mathcal{S}_0^i}{\partial v} \delta v + \frac{\partial \mathcal{S}_0^i}{\partial h} \delta h \\ &= \left(\frac{\partial f^{-1}(u, v)}{\partial u} + \eta h \frac{\partial \mathbf{n}(f^{-1}(u, v), \mathcal{S})}{\partial u} \right) \delta u \\ &\quad + \left(\frac{\partial f^{-1}(u, v)}{\partial v} + \eta h \frac{\partial \mathbf{n}(f^{-1}(u, v), \mathcal{S})}{\partial v} \right) \delta v + (\eta \mathbf{n}) \delta h. \end{aligned} \quad (18)$$

In the above equation, $\eta \in [0, 1]$ denotes the dimensionless local integration coordinate along the stiffener height direction. Acting as an independent parametric variable, it undergoes no variation (i.e., $\delta \eta \equiv 0$). Eq. (18) demonstrates the completeness of the geometric mapping framework: theoretically, both the planar position (u, v) and the spatial height h of the stiffeners can serve as independent design variables for synergistic optimization. In the present study, the height h of all

stiffeners is assumed to be a prescribed global constant; consequently, it is not updated during the optimization process (i.e., $\delta h = 0$). It should be noted that this assumption reduces the dimensionality of the design problem, thereby helping to focus on the evolution of explicit layouts on complex curved surfaces. Fixing the stiffener height reduces the dimensionality of the problem but also narrows the feasible design space and may limit the attainable optimum. Variable-height optimization is not implemented in the present study and is left for future work.

Therefore, by leveraging the Lagrange interpolation relationship inherent in the adaptive ground structure method, the evolutionary velocity of any arbitrary point (u, v) within the parameter domain can be expressed as a linear interpolation driven by the two endpoint nodes P_1 and P_2 . Specifically, this is formulated as $\delta u = (1 - \mu)\delta u_1 + \mu\delta u_2$ (with an analogous expression for δv). By substituting these skeleton endpoint variations back into Eq. (18), and subsequently back-substituting the results sequentially into Eqs. (17) and (15), one ultimately obtains the exact analytical partial derivatives of the objective function with respect to the system's global design variables $\mathbf{D} = (\mathbf{D}_p^\top, \mathbf{D}_t^\top)^\top$.

4. Numerical implementation and stabilization strategies

4.1. Finite element discretization and sensitivity evaluation

In the numerical implementation of the proposed framework, to balance geometric fidelity to complex base surfaces with computational efficiency, the base skin is discretized using three-node triangular general-purpose shell elements (Abaqus S3). Conversely, for the stiffener components, which possess explicit topology and well-defined boundaries, four-node quadrilateral reduced-integration shell elements (Abaqus S4R) are employed for independent meshing. During the iterative process of explicit topology optimization, the positions and morphologies of the stiffeners are in a state of continuous evolution. To avoid the computationally prohibitive cost of conformal remeshing after every geometric update, this paper adopts a non-matching mesh discretization strategy. Specifically, by leveraging the built-in surface-based tie constraint capability of the Abaqus solver, the degrees of freedom (DOFs) of the independently meshed stiffener bases (defined as the slave surfaces) are coupled to those of the base skin (defined as the master surfaces) via multi-point constraint equations.

Notably, this conventional finite element treatment aligns well with the geometric characteristics of the explicit topology optimization framework proposed herein. In traditional implicit topology optimization methods (such as SIMP or certain level-set methods), capturing distinct thin-walled stiffeners in three-dimensional space typically necessitates a high-resolution background mesh, which often leads to a significant increase in the total number of DOFs. In contrast, since our method employs explicit analytical geometric boundaries to describe the stiffener components, it allows the base skin and individual components to undergo appropriate independent discretization without being heavily dependent on the global mesh density. This strategy effectively alleviates the difficulties associated with mesh reconstruction during the component shape evolution process. By maintaining high geometric fidelity while keeping the system DOFs within a reasonable range, it provides a computationally friendly implementation pathway for the dynamic optimization of large-scale complex curved stiffened structures.

In the context of discrete sensitivity analysis, for Eq. (17), let $\mathcal{B}_0 = \bigcup_{e=1}^{N_0} \mathcal{B}_0^e$, where $\mathcal{B}_0^e$ denotes the e -th shell element of the base skin. The discretized stiffener region is expressed as $\mathcal{B}_i(\mathbf{D}) = \bigcup_{e=1}^{N_i} \mathcal{B}_i^e(\mathbf{D}) = \bigcup_{e=1}^{N_i} (\mathcal{S}_i^e(\mathbf{D}) \times [-t_i/2, t_i/2])$, where $\mathcal{B}_i^e(\mathbf{D})$ represents the e -th shell element on the i -th stiffener (see Fig. 8). For implementation purposes and to simplify notation, the symbol d is employed here to represent an arbitrary component of the design variable vector $\mathbf{D}$ that controls the stiffener geometry. The discrete sensitivity is thus given by:

$$\frac{\partial \lambda}{\partial d} = \sum_{i=1}^{n_c} \sum_{e=1}^{N_i} \left(W(\mathbf{U})_e^{\text{upper}} + W(\mathbf{U})_e^{\text{lower}} \right) A_e, \quad (19)$$

where $W(\mathbf{U}) = (\mathbf{E} : \boldsymbol{\varepsilon}(\mathbf{U}) : \boldsymbol{\varepsilon}(\mathbf{U}) - \lambda \rho \mathbf{U} \cdot \mathbf{U}) \mathbf{V}_n^t$. The notations $(\bullet)_e^{\text{upper}}$ and $(\bullet)_e^{\text{lower}}$ denote the values of the physical quantity $(\bullet)$ evaluated at $\mathcal{B}_i^e(\mathbf{D})|_{\zeta=1/2}$ and $\mathcal{B}_i^e(\mathbf{D})|_{\zeta=-1/2}$, respectively (only the top and bottom surfaces of $\mathcal{B}_i^e$ need to be considered, which correspond to the lateral boundaries of the stiffener). Here, A_e represents the area of the quadrilateral mesh element $\mathcal{B}_i^e(\mathbf{D})$.

It is noteworthy that in Eq. (19), λ represents a non-repeated fundamental eigenvalue. To address the issue of repeated eigenvalues in structural optimization, the solution strategy proposed by Gravesen et al. [47] is adopted: when dealing with m repeated eigenvalues $\tilde{\lambda} = \lambda_j$ ($j = 1, \dots, m$), the sensitivities are calculated by summing the derivatives of each individual eigenvalue with respect to the design variables, i.e.,

$$\frac{\partial \tilde{\lambda}}{\partial d} = \sum_{j=1}^m \frac{\partial \lambda_j}{\partial d} \quad (20)$$

To further clarify the numerical criterion for identifying repeated or nearly repeated eigenvalues, a relative eigenvalue-gap tolerance is introduced in this study. This treatment follows the idea of using a relative-difference tolerance to determine the effective multiplicity of eigenvalues in structural eigenvalue optimization, as adopted by A. P. Seyranian et al. [48]. Specifically, for the low-order eigenvalues under consideration, λ_i ($i = 1, \dots, m$), their relative gaps with respect to the first eigenvalue λ_1 are computed as

$$\delta_i = \frac{|\lambda_i - \lambda_1|}{|\lambda_1|}, i = 1, \dots, m. \quad (21)$$

When $\delta_i \leq \varepsilon$, the corresponding eigenvalue λ_i is regarded as belonging to the same nearly repeated eigenvalue cluster as λ_1 . In this work, $\varepsilon = 10^{-4}$ is adopted as the eigenvalue-gap tolerance. This threshold is used only to identify whether a low-order eigenvalue cluster occurs and to determine whether the summation-type sensitivity stabilization in Eq. (20) should be activated. If no nearly repeated eigenvalue cluster is detected, the single-eigenvalue sensitivity in Eq. (19) is still used.

The summation strategy in Eq. (20) characterizes the overall variation trend of the repeated eigenvalue cluster and can therefore provide more stable sensitivity information when the fundamental frequency is governed by a cluster of low-order eigenvalues. It should be emphasized that Eq. (20) is used only as a numerical stabilization strategy for repeated or nearly repeated eigenvalues. It does not imply that the optimization objective is changed from maximizing the first eigenvalue to maximizing the sum of eigenvalues in the cluster. The optimization objective in this study remains the first eigenvalue throughout. Accordingly, objective evaluation and convergence assessment at each iteration are both based on the current first eigenvalue. The frequencies reported and compared in the final results are also the first natural frequencies, rather than the sum of the eigenvalues within the cluster.

4.2. Thickness–density double penalty scheme for suppressing local modes

When applying explicit topology optimization (particularly methods incorporating a component removal mechanism) to fundamental frequency maximization, a critical numerical challenge arises: the generation of spurious local modes. This issue has also been widely discussed in eigenfrequency topology optimization. For example, Pedersen [49] proposed a modified interpolation scheme to alleviate artificial localized vibration modes associated with low density elements in density-based topology optimization. Different from such density field based treatments, the present study focuses on localized modes induced by the degradation and removal of explicit stiffener components in an MMC-based stiffened shell optimization framework. Therefore, the

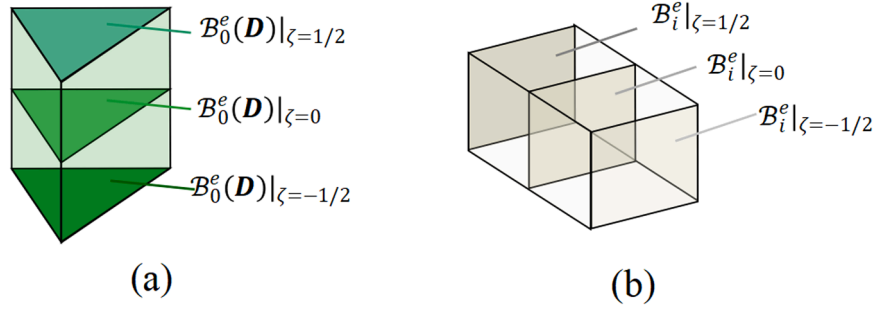


Fig. 8. Schematic representation of the finite element discretization: (a) three-node S3 elements for the base skin; (b) four-node S4R elements for the stiffeners.

proposed method adopts a new treatment to avoid the localized mode issue by coupling the stiffener thickness variable with the component-removal criterion through a Thickness–Density Double Penalty Scheme, so that weakened stiffeners approaching the deletion threshold can be smoothly excluded from dominating the low-order frequency response.

As the thickness t of a specific stiffener gradually approaches zero during the optimization process, shell element theory dictates that its out-of-plane bending stiffness matrix $\mathbf{K}_e$ is proportional to the cube of the thickness ($\mathbf{K}_e \propto t^3$), whereas its mass matrix $\mathbf{M}_e$ is proportional only to the first power of the thickness ($\mathbf{M}_e \propto t$).

This discrepancy in decay rates results in the local structural frequency $\omega_{\text{local}} \propto \sqrt{\mathbf{K}_e/\mathbf{M}_e} \propto t$. Obviously, when the thickness is extremely small, the local natural frequency will drop sharply and intrude into the low-order global mode region, potentially forming spurious low-frequency local vibrations with extremely high amplitudes. This not only severely interferes with the calculation of global gradients but can even lead to the collapse of the optimization iteration.

To fundamentally resolve this phenomenon from both physical and mathematical perspectives, a Thickness–Density Double Penalty Scheme is proposed. First, to satisfy manufacturing constraints and implement component removal smoothly, a thickness penalty mechanism based on a translated Heaviside function is introduced:

$$t_i^{\text{pen}} = H_\epsilon^a(t_i - t_l)t_i, \quad (22)$$

where t_i^{pen} denotes the penalized thickness, and the corresponding regularized Heaviside function $H_\epsilon^a(x)$ is defined with respect to $x = t_i - t_l$ as:

$$H_\epsilon^a(x) = \begin{cases} 1, & x > \epsilon \\ \frac{3(1-\alpha)}{4} \left(\frac{x}{\epsilon} \right)^3 + \frac{1+\alpha}{2}, & |x| \leq \epsilon \\ \alpha, & x < -\epsilon \end{cases} \quad (23)$$

Here, t_l is the prescribed lower bound constraint on thickness, ϵ is a parameter controlling the regularization width, and α is a small positive number used to set the threshold t_r for the stiffener removal mechanism, defined as $\alpha = t_r/(t_l - \epsilon)$. In practical optimization, the original thickness of the i -th stiffener, t_i is replaced by t_i^{pen} . By applying the Heaviside penalty function, as shown in Fig. 9, only for components satisfying $t_i \in [t_l - \epsilon, t_l + \epsilon]$ will the penalized thickness t_i^{pen} fall within the interval $[t_r, t_l + \epsilon]$. Correspondingly, by setting the parameter ϵ to a small value, the number of stiffeners with penalized thickness $t_i^{\text{pen}} \in [t_r, t_l + \epsilon]$ is further reduced, thereby achieving effective control over the minimum thickness of the stiffeners.

Relying solely on the thickness penalty is insufficient to resolve the stiffness-to-mass mismatch problem. Therefore, the core step of this mechanism is to impose a higher-order penalty on the material density of those weak components whose thickness approaches the removal threshold:

$$\rho_i^{\text{pen}} = [H_\epsilon^a(t_i - t_l)]^p \rho, \quad (24)$$

where ρ_i^{pen} denotes the penalized equivalent material density, and the penalty factor p is a real-valued penalty exponent greater than 2 ($p = 3$ in this study). Under this dual penalty mechanism, the effective mass of the weakened stiffeners decays at a faster rate than their effective stiffness (i.e., $\mathbf{M}_e^{\text{pen}} \propto t^p \cdot t = t^{p+1}$). Consequently, the local eigenfrequency transforms to $\omega_{\text{local}} \propto \sqrt{t^3/t^{p+1}}$. When $p > 2$ is selected, as the thickness $t \rightarrow 0$, the penalized local frequency tends towards infinity ($\omega_{\text{local}} \rightarrow \infty$).

The mechanisms described above regularize the contribution of extremely thin stiffeners approaching removal by assigning them a modified stiffness-to-mass relation. Consequently, in modal analysis, the influence of spurious local modes associated with these weakened stiffeners is reduced in the low-order frequency range of interest. This penalty strategy, based on the decoupling of physical mechanisms, helps enforce the minimum thickness constraints and improves the numerical robustness of the fundamental frequency evolution process.

4.3. Optimization algorithm and solution procedure

Integrating the explicit geometric description, non-matching finite element discretization strategy, and analytical sensitivity derivation discussed previously, a comprehensive numerical solution framework for the dynamic topology optimization of complex curved stiffened structures is established. The procedure is summarized in Fig. 10 and includes the following steps:

- (1) **Algorithm Initialization and Pre-processing:** Input the geometric parameters of the base surface, material properties, and boundary conditions. Discretize the base skin using a fixed shell element mesh. Crucially, establish the conformal mapping between the 2D parametric domain and the 3D base surface *once and for all*. Initialize the global design variables $\mathbf{D}^{(0)}$ to specify the quantity and layout of the stiffener components. Utilizing the established conformal mapping, generate the initial 3D physical configurations of the stiffeners and independently discretize each stiffener component. The mesh connectivity of both the base skin and the individual stiffeners will remain constant throughout subsequent iterative steps, which serves as a fundamental guarantee of computational efficiency.
- (2) **Geometry Update and Kinematic Assembly:** Based on the updated design variables $\mathbf{D}^{(k)}$ at the current iteration, update the spatial coordinates of the stiffener mesh nodes in 3D space using the pre-established conformal mapping, *without generating any new meshes*. Subsequently, execute the non-matching mesh assembly by coupling the degrees of freedom of the explicitly updated stiffener bases (slave surfaces) to the fixed base skin (master surface) via the surface-based tie constraints built into the finite element solver.

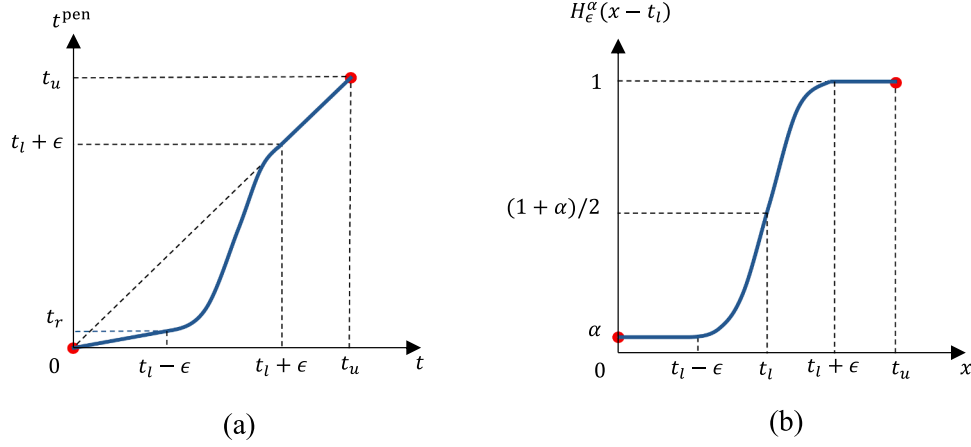


Fig. 9. Thickness penalty based on the Heaviside function: (a) Penalized thickness t_i^{pen} ; (b) Penalty function H_ϵ^α .

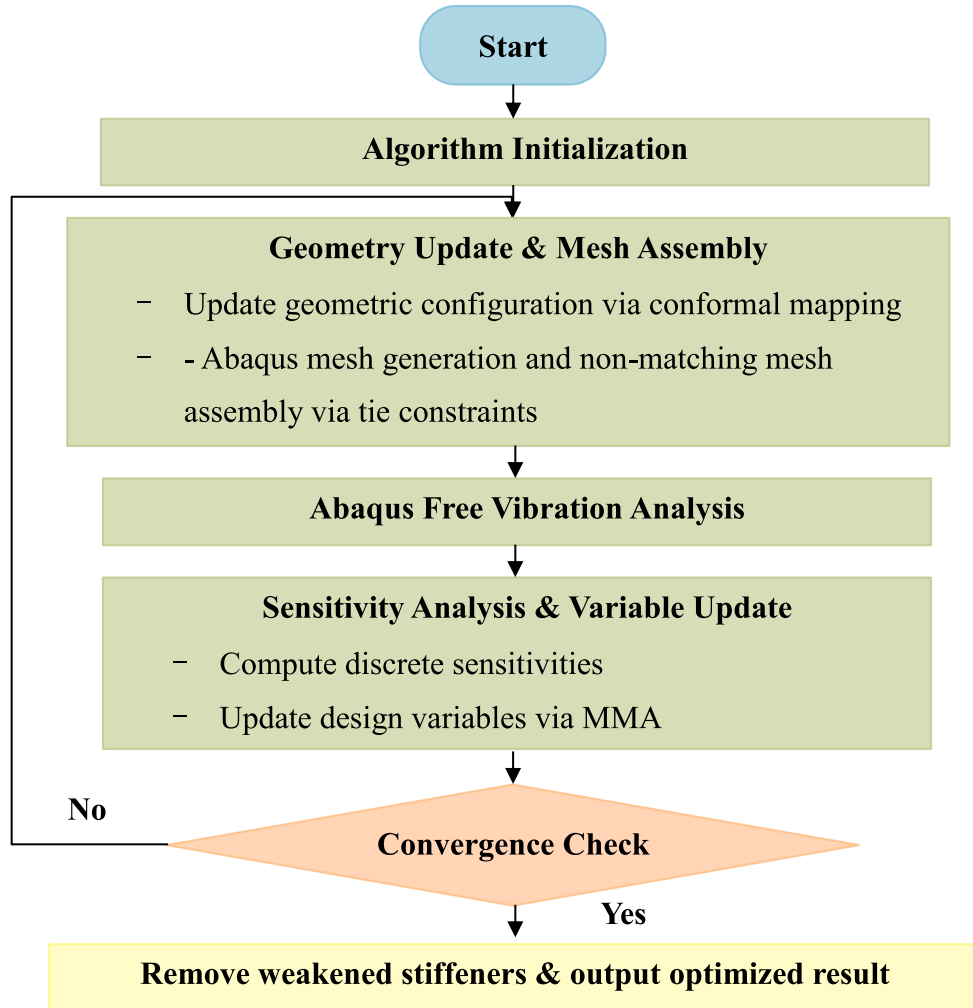


Fig. 10. Flowchart of the Optimization Procedure.

(3) **Free Vibration Analysis:** Invoke the finite element solver to construct the global stiffness matrix and mass matrix of the assembled structural system. The proposed Thickness-Density Double Penalty Scheme is simultaneously applied to physical properties. Solve the generalized undamped free vibration

eigenvalue problem to extract the fundamental frequency of the structure and the corresponding global mode shape vectors.

(4) **Sensitivity Analysis and Variable Update:** Based on the aforementioned finite element responses, calculate the discrete sensitivities according to Eqs. (19) and (20). The Method of

Moving Asymptotes (MMA) is then employed to iteratively update the design variables $\mathbf{D}$. During the optimization process, the topology and geometry of the structure evolve continuously, which may lead to mode switching. To identify the correspondence between the dominant mode shapes in two adjacent iterations and to assist in checking whether mode switching occurs, the Modal Assurance Criterion (MAC) reported in Refs. [50,51] is adopted in this study as an auxiliary modal identification measure. The MAC is used as an a posteriori diagnostic of modal consistency between adjacent iterations; it is not used to redefine the objective, enforce a fixed modal identity, or compute the eigenvalue sensitivity. The optimizer always uses the current lowest non-rigid eigenvalue.

- (5) **Convergence Check:** Evaluate the convergence criteria for the current iteration. The optimization process is deemed converged when the relative change in the objective function value remains below a prescribed tolerance for a consecutive number of steps, or when the maximum allowable number of iterations is reached. Upon convergence, output the final explicit geometric configuration of the curved stiffened structure. Otherwise, update the iteration counter ($k = k + 1$) and return to Step (2) to initiate the next cycle.

5. Numerical examples

This section systematically validates the effectiveness, convergence robustness, and performance in representative numerical examples of the proposed explicit topology optimization method for fundamental frequency maximization in complex thin-walled stiffened structures. The validation is conducted through four typical numerical examples with progressively increasing geometric complexity. In all cases, the design variables are updated using the Method of Moving Asymptotes (MMA). The geometric configurations of these examples evolve from a basic planar surface to free-form surfaces with negative Gaussian curvature, further to closed cylindrical shells, and ultimately to an aerospace assembly structure comprising multiple connected domains. This progression allows for a comprehensive evaluation of the method's optimization performance on complex manifolds. To distinguish from the natural frequency parameter λ discussed earlier, the first-order fundamental natural frequency in this section and subsequent figures is uniformly denoted as Ω_1 (unit: Hz).

5.1. Square plate

To validate the effectiveness of the proposed explicit stiffener optimization framework and establish a baseline for comparison, we first consider the fundamental frequency optimization of a four-point simply supported square plate, as illustrated in Fig. 11(a). Referencing the literature [36], the geometric model and material properties are defined as follows: the base skin and stiffeners share an identical elastic modulus of $E_p = E_s = 206$ GPa, a Poisson's ratio of $\nu_p = \nu_s = 0.3$, and a density of $\rho = 7800$ kg/m³. The base skin thickness is fixed at $t_p = 0.2$ mm. In this example, the stiffener height is constant at $h_s = 2$ mm, while the continuous thickness is adopted as the design variable. Starting from an initial value of $t_0 = 0.5$ mm, the thickness search interval is bounded within $t \in [0.0001, 1]$ mm, with a stiffener removal threshold of $t_r = 0.0004$ mm. Fig. 11(b) depicts the initial first-order mode shape and fundamental frequency of the unstiffened bare shell. The optimization objective is to maximize the structure's first-order natural frequency, subject to the constraint that the total stiffener volume does not exceed 0.6 times the base skin volume (i.e., a prescribed volume fraction upper limit of $\bar{V} = 0.6$).

To validate the suppression effect of the Thickness-Density Double Penalty Scheme (proposed in Section 4.2) on spurious local modes, this example establishes two schemes for a rigorous comparison: Scheme I

applies only the pure thickness penalty to the stiffeners using Eq. (22), while Scheme II additionally imposes a synergistic penalty on the material density of weakened stiffeners based on Eq. (24). For visualization purposes, the stiffener thicknesses in all subsequent figures are magnified by a factor of 4. Fig. 12 presents the initial structural configuration and its corresponding first-order mode shapes when employing a 6×6 initial stiffener grid as the starting layout.

The comparative optimization results for the two schemes are presented in Table 5.1. In the figures, the line thickness intuitively reflects the final thickness distribution of the stiffeners, while the thin black lines represent "weakened stiffeners" whose thickness has approached the lower dimensional limit and is on the verge of triggering the removal mechanism.

From the perspective of modal evolution behavior, significant differences exist between the results of the two schemes. As shown in Table 5.1 and the iteration curves in Fig. 13, the first scheme (unidirectional thickness penalty) fell into severe "spurious local modes" as early as the initial iteration stage (Step 4). This caused the fundamental frequency of the system to drop to a level lower than that of the initial design (44.75 Hz), resulting in oscillations in the iteration curve.

In contrast, the second scheme, which employs the dual penalty mechanism, reduces the influence of low-energy local vibration modes associated with extremely thin stiffeners by decoupling their stiffness and mass contributions during the removal process. This scheme promotes a more stable reconfiguration of the stiffener layout and leads to a converged fundamental frequency of 74.50 Hz. After extracting the final physical layout of Scheme II by removing weak stiffeners, the recalculated fundamental frequency is 74.57 Hz, indicating that the extracted layout preserves the dynamic performance of the continuous optimization result. This comparison suggests that localized modes associated with weakened stiffeners can interfere with fundamental-frequency optimization, and that the proposed dual penalty mechanism helps mitigate this issue. Based on this observation, the dual penalty strategy is adopted as the default approach for subsequent complex curved-surface examples.

To assess the computational efficiency of the proposed framework, the representative 6×6 square-plate example was used as a benchmark. The computations were carried out on a workstation with an Intel Core i9-13900K CPU and 64 GB RAM. Completing 1000 optimization iterations required approximately 39828 s in total. The time spent on pre-processing, input-file generation, finite element analysis, and result post-processing was approximately 93 s, 7852 s, 23009 s, and 8746 s, respectively. These statistics show that the overall computational cost is mainly associated with finite element analysis and result post-processing. They also indicate that the repeated geometry updates, generation of analysis files, and extraction of numerical results are the major time-consuming steps in the current implementation. For larger-scale applications, the efficiency of the framework may be improved by parallel finite element analysis, faster input-file generation, and optimized result-reading strategies.

To investigate the influence of the initial grid topology on the final configuration, this example further compares two additional initial single-cell arrangements with different densities: a 4×4 initial stiffening grid and an 8×8 initial stiffening grid, as illustrated in Fig. 14. Table 5.2 details the structural models, mode shape characteristics, and fundamental frequency evolution at various stages. It can be observed that as the density of the initial stiffening grid increases, the total mass and stiffness distribution of the initial structure change, thereby leading to variations in the initial fundamental frequency.

After topology optimization, all initial base structures evolved into "primary stiffener skeletons" that align closely with the strain energy distribution of the dominant mode. Compared to the unstiffened state, the fundamental frequency increases substantially across all cases. Although differences persist in local topological details and the maximum thickness distribution, the macroscopic extension directions of the principal stiffeners show a similar trend.

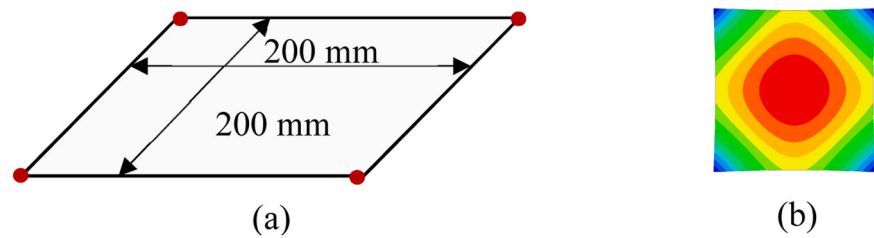


Fig. 11. Geometric model and initial first-order mode shape of the square plate: (a) Geometric model; (b) First-order mode shape of the unstiffened shell ($\Omega_1 = 8.80$ Hz).

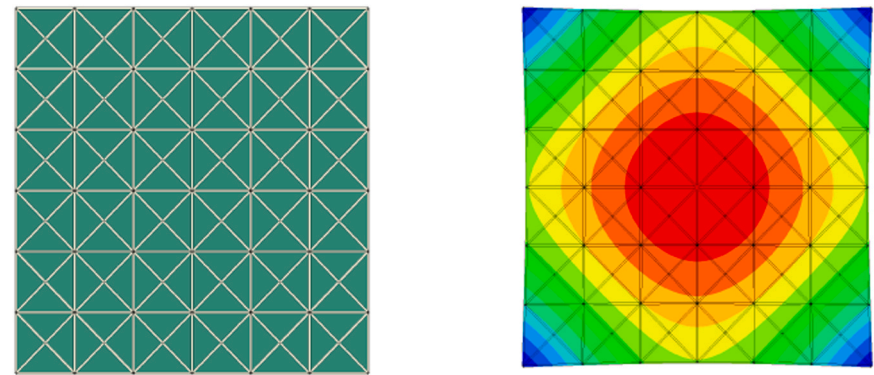


Fig. 12. Initial design with a 6×6 stiffener grid and its corresponding first-order mode shape ($\Omega_1 = 53.65$ Hz).

Table 5.1
Optimization results and modal analysis of a four-point simply supported stiffened plate under two penalty schemes.

		Scheme I		Scheme II	
		Continuous Layout	Extracted Layout	Continuous Layout	Extracted Layout
Iteration 4	Stiffener layout				
	First-order mode shape	 $\Omega_1 = 44.75$ Hz	 $\Omega_1 = 69.48$ Hz	 $\Omega_1 = 69.69$ Hz	 $\Omega_1 = 69.46$ Hz
Iteration 1000	Stiffener layout				
	First-order mode shape	 $\Omega_1 = 67.17$ Hz	 $\Omega_1 = 69.37$ Hz	 $\Omega_1 = 74.50$ Hz	 $\Omega_1 = 74.57$ Hz

The physical essence of this phenomenon lies in the non-penetration constraints imposed on the motion domain of driving nodes to ensure sharp explicit geometric boundaries and prevent stiffener intersection or overlap. While this restriction limits the cross-domain search for an absolute global optimum, introducing a degree of path dependence on the initial base structure, it conversely grants the method significant freedom for engineering exploration. As illustrated by the iteration curves in Fig. 15 and the final data in Table 5.2, although the three

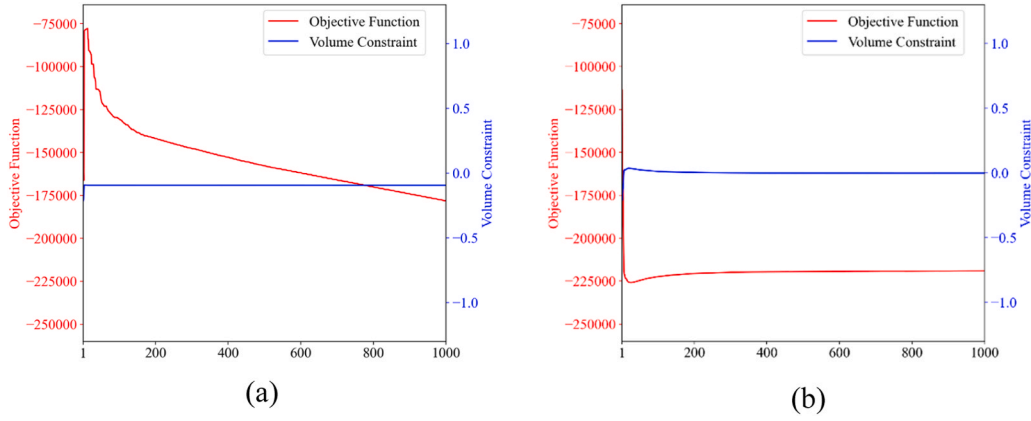


Fig. 13. Optimization iteration curves (continuous layout) for the two penalty schemes under a 6×6 initial stiffener grid: (a) Thickness-only penalty; (b) Dual penalty mechanism.

initial stiffening grids (4×4 , 6×6 , and 8×8) evolve into distinct topological morphologies, their limiting fundamental frequencies after removing weakened stiffeners (75.01 Hz, 74.57 Hz, and 74.50 Hz, respectively) exhibit a maximum relative deviation of less than 1%. This performance-equivalent multi-configuration redundancy demonstrates that the proposed method can provide multiple alternative stiffening configurations for the engineering conceptual design phase. These configurations show comparable first-order natural frequencies and distinct stiffener layout patterns, while maintaining small performance fluctuation.

Because Ref. [36] does not report detailed information on all stiffener sections, mesh discretization, design variables, and final structural volume, the objective function values of the optimized results obtained by different methods cannot be directly compared. Under a square-plate problem setting close to that of the benchmark in Ref. [36], the extracted 6×6 layout in this study reaches a first natural frequency of 74.57 Hz and provides an explicitly parameterized stiffener configuration.

5.2. Saddle surface

To verify the effectiveness and geometric conformity of the CCM technique in stiffener optimization for complex curved shells, this section investigates a saddle-shaped shell model characterized by typical negative Gaussian curvature. As shown in Fig. 16(a), the saddle surface is generated by sweeping two circular arcs that share a common vertex and lie in mutually perpendicular planes. Clamped boundary conditions are applied at the four corner points of the shell (indicated by the red regions in Fig. 16(b)). The base skin thickness of the shell is $t_p = 1$ mm. The elastic modulus, Poisson's ratio, and density for both the base skin and the stiffeners are $E_p = E_s = 206$ GPa, $\nu_p = \nu_s = 0.3$, and $\rho_p = \rho_s = 7800$ kg/m³, respectively. The height of the stiffeners is fixed at $h_s = 5$ mm, while their continuous thickness serves as the design variable for optimization. The initial thickness is set to $t_0 = 1$ mm. During the

optimization process, the search range for thickness is constrained to $t \in [0.1, 2]$ mm, with a stiffener removal threshold of $t_r = 0.15$ mm. Fig. 17 presents the initial fundamental mode shape and the corresponding natural frequency of the four-point clamped saddle shell. The optimization objective for this example is to maximize the fundamental natural frequency of the structure, subject to the constraint that the total volume of the stiffeners does not exceed half of the base skin volume (i. e., an upper bound on the volume fraction of $\bar{V} = 0.5$).

As shown in Fig. 18, the initial skeletal layout employs an 8×6 initial stiffener grid, comprising a total of 302 explicit stiffener components and 492 design variables. Visual inspection of the initial configuration shows that, despite the negative Gaussian curvature of the base surface, the generated explicit stiffener entities maintain good geometric conformity and normal offset relative to the base surface. This is attributed to the angle-preserving properties of the CCM technique employed. These results demonstrate the capability of the proposed framework to handle stiffener modeling on curved manifolds with pronounced curvature variations.

Fig. 19 illustrates the continuous thickness distribution of results after optimization and the extracted physical layout following the removal of weakened stiffeners. The corresponding fundamental natural frequency is 294.98 Hz for the continuous layout and 294.57 Hz for the extracted-layout model, with a relative difference of approximately 0.14%. Observation of the optimization results reveals that the stiffener layout underwent significant adaptive reconstruction compared to the initial grid, leading to a substantial leap in the fundamental natural frequency compared to the unstiffened shell.

Notably, in the central region of the extracted saddle shell layout, a phenomenon of disconnected stiffener segments is observed. This is precisely attributed to the thickness-density dual penalty scheme introduced earlier, which adaptively weakens and eliminates continuous stiffener segments that contribute negligibly to the overall fundamental frequency. In terms of modal evolution, the remaining discrete

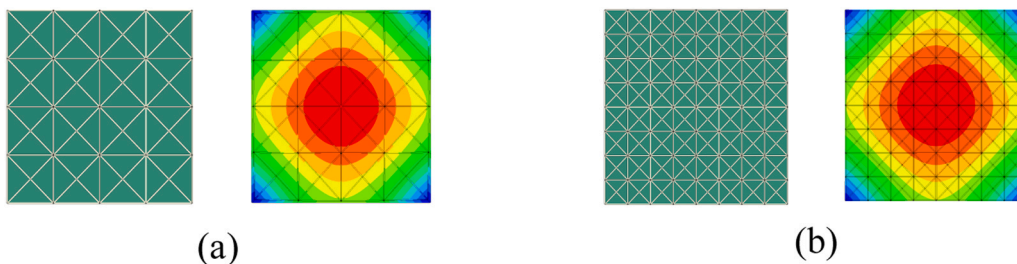
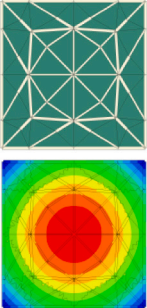
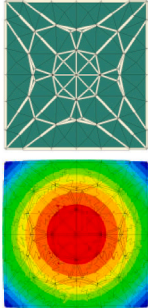
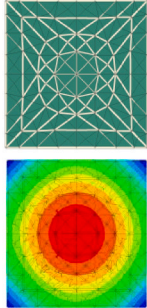
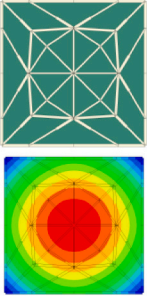
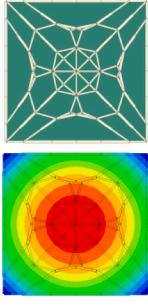
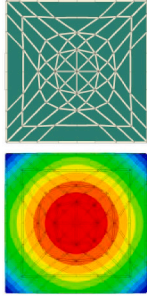


Fig. 14. Designs of two initial stiffening grids with different densities and their corresponding first-order mode shapes: (a) 4×4 initial stiffener grid ($\Omega_1 = 53.65$ Hz); (b) 8×8 initial stiffener grid ($\Omega_1 = 57.71$ Hz).

Table 5.2
Structural configurations and modal analysis of the four-point simply supported stiffened plate under different initial grids.

	4×4 initial stiffener grid	6×6 initial stiffener grid	8×8 initial stiffener grid
Continuous Layout	 $\Omega_1 = 74.00 \text{ Hz}$	 $\Omega_1 = 74.50 \text{ Hz}$	 $\Omega_1 = 74.23 \text{ Hz}$
Extracted Layout	 $\Omega_1 = 75.01 \text{ Hz}$	 $\Omega_1 = 74.57 \text{ Hz}$	 $\Omega_1 = 74.50 \text{ Hz}$

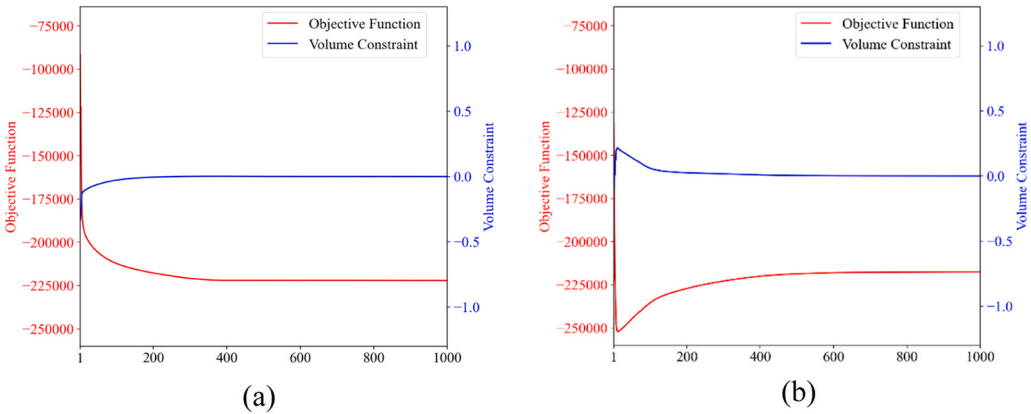


Fig. 15. Optimization iteration curves (continuous layout) for two initial stiffener grid designs: (a) Iteration history for the 4×4 initial stiffener grid; (b) Iteration history for the 8×8 initial stiffener grid.

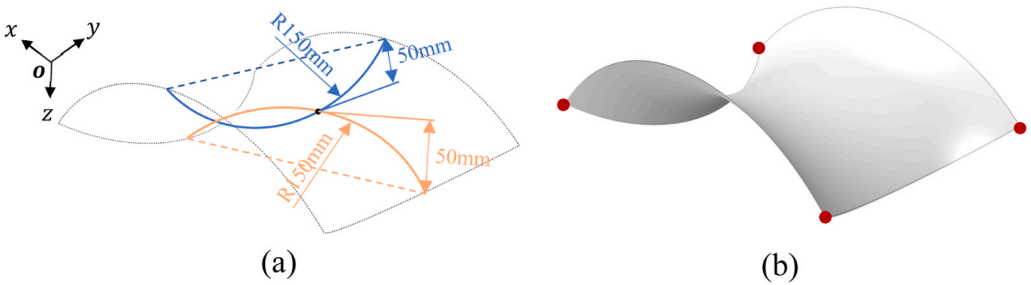


Fig. 16. Geometric model and boundary conditions of the saddle shell: (a) Geometric model; (b) Boundary conditions.

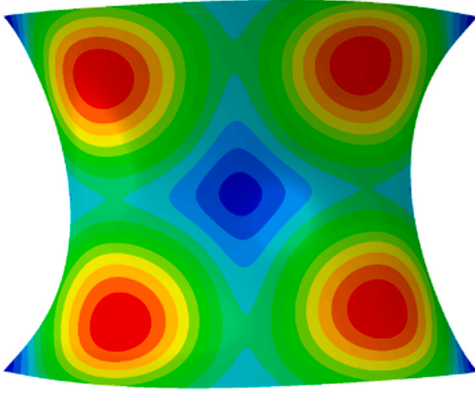


Fig. 17. Initial fundamental mode shape and natural frequency of the saddle shell ($\Omega_1 = 173.30$ Hz).

stiffeners essentially function as local stiffness/mass tuners, suppressing local amplitudes at a minimal mass cost. The close agreement between the fundamental frequency values before and after the removal of these inefficient stiffeners (294.98 Hz vs. 294.57 Hz) supports the physical rationality of this phenomenon and the effectiveness of the removal mechanism.

The optimization iteration curves in Fig. 20 clearly reveal the two-stage evolutionary characteristics of this process. In the initial stage, the fundamental frequency rises rapidly. This is driven by the penalty mechanism, which forces the stiffener thicknesses in high strain energy regions (critical areas) to quickly approach their upper constraints. Once the thicknesses in these key regions hit their limits, the iteration curve reaches an inflection point. Subsequently, further improvements in eigenvalues rely primarily on the collaborative movement of explicit driving nodes within the parametric domain.

5.3. Irregular nozzle-like curved shells

To validate the effectiveness of the proposed surface cutting and mapping techniques in handling closed manifold boundaries, this section considers a complex cylindrical-like shell structure. The study focuses on the irregular nozzle-like shell model shown in Fig. 21(a), where

the boundary of the right cylindrical segment is fully clamped (indicated by the red region in Fig. 21(b)). The base shell thickness is $t_b = 2$ mm. The elastic modulus, Poisson's ratio, and density for both the base skin and the stiffeners are $E_p = E_s = 70$ GPa, $\nu_p = \nu_s = 0.33$, and $\rho_p = \rho_s = 2700$ kg/m³, respectively. The stiffener height is fixed at $h_s = 5$ mm, with the continuous thickness serving as the design variable for optimization. The initial thickness is set to $t_0 = 0.5$ mm, with the search space constrained to $t \in [0.01, 2]$ mm and a stiffener removal threshold of $t_r = 0.05$ mm. Fig. 22 presents the initial first-order mode shape of the unstiffened nozzle shell, yielding a fundamental frequency of 785.83 Hz. The upper bound constraint for the volume fraction in this example is set to $\bar{V} = 0.5$ (i.e., the total volume of the stiffeners must not exceed 0.5 times the volume of the base shell).

To achieve efficient parameterization of such closed cylindrical models, the surface cutting strategy introduced in Fig. 7(a) is employed, as illustrated in Fig. 23. The closed shell is cut along its generatrix and unfolded into a parameter domain with a simply connected topology. The open-surface parameter domain is used to organize the reference positions and connectivity of the explicit stiffeners, while the corresponding stiffener geometry is generated on the physical nozzle surface through the established mapping relation for finite element analysis. To prevent material overlap of stiffeners at the common boundaries, the algorithm automatically eliminates redundant components located at one side of the seam during the mapping and seaming phase.

It is particularly important to note that, since the driving nodes of the explicit stiffeners are strictly confined to move within their corresponding parameter domains, the stiffeners located at the seam in the physical space (i.e., the common boundary) must be realized by driving nodes that lie exactly on the parametric boundary to ensure geometric continuity and end-to-end connectivity. Furthermore, throughout the entire optimization iteration process, the sliding trajectories of these common nodes are strictly constrained to the seam boundary, thereby helping preserve the topological integrity of the closed cylindrical shell.

Fig. 24 illustrates the final physical layout of the stiffened irregular nozzle shell. The results show that even in the transition regions characterized by large curvature gradients, the optimized stiffener skeleton maintains good conformity with the base skin. This further demonstrates the effectiveness of the proposed mapping method in handling complex closed surfaces. The corresponding steady iteration convergence curve is

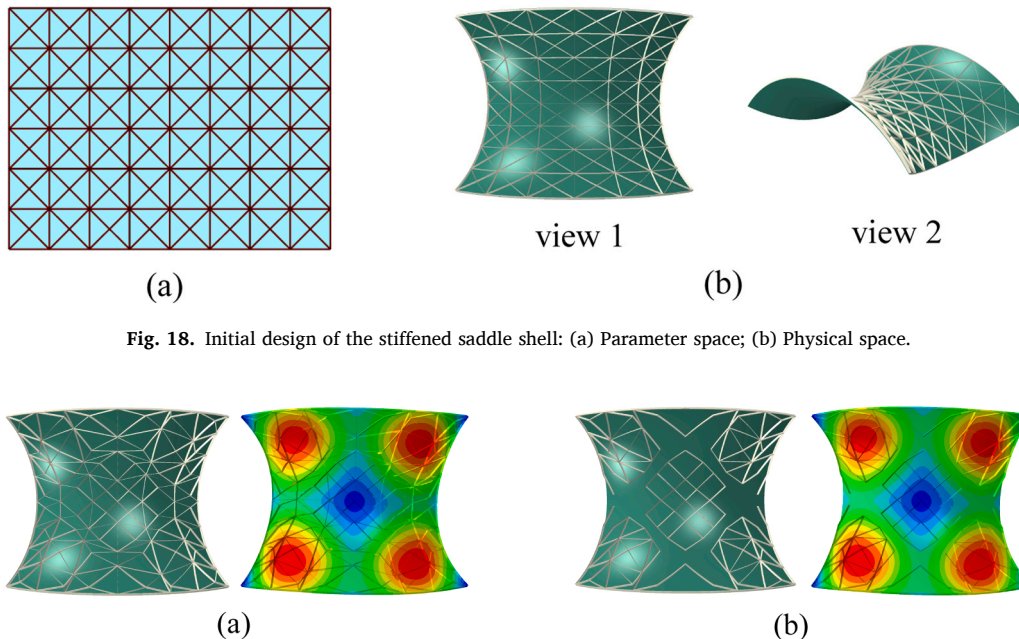


Fig. 18. Initial design of the stiffened saddle shell: (a) Parameter space; (b) Physical space.

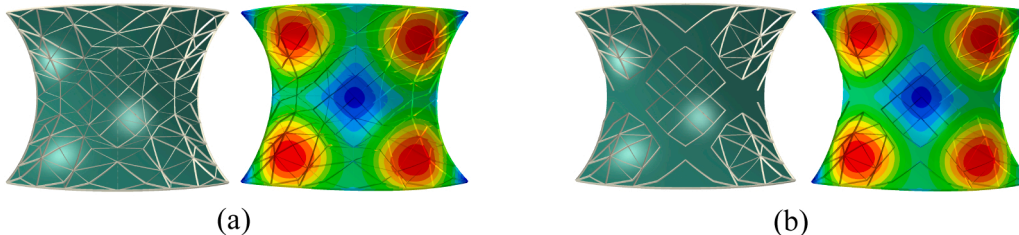


Fig. 19. Continuous layout and extracted layout of the stiffened saddle shell: (a) Continuous layout ($\Omega_1 = 294.98$ Hz); (b) Extracted layout ($\Omega_1 = 294.57$ Hz).

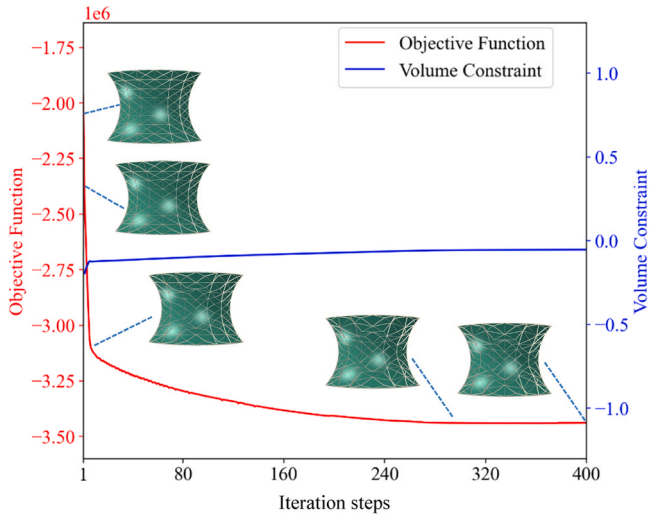


Fig. 20. Optimization iteration curves (continuous layout) and intermediate results of the stiffened saddle shell, with $\Omega_1^1 = 223.13$, $\Omega_1^2 = 259.49$, $\Omega_1^6 = 278.71$, $\Omega_1^{300} = 294.68$, $\Omega_1^{400} = 294.98$ (Ω_1^n represents the fundamental frequency value at the n th iteration step).

presented in Fig. 25.

Benefiting from the strong geometric interpretability and design flexibility inherent in the explicit MMC stiffener optimization framework, the proposed method allows selected geometric constraints to be readily imposed. For instance, in practical design workflows, to reduce complexity and enhance manufacturability, it is often necessary to strictly limit the number of stiffeners intersecting at any single node. To address such process requirements, this issue can be effectively avoided by configuring an orthogonal initial stiffener grid without diagonal members, as illustrated in Fig. 26.

Furthermore, to achieve configurations with clearer stiffener connectivity and improved layout regularity, a coupled nodal-position constraint can be imposed on members within the same transverse (or longitudinal) family, building upon the aforementioned orthogonal grid. Specifically, this involves strictly synchronizing the coordinate updates of associated nodes along the tangential direction, which effectively realizes the spatial coupling and binding of design variables in that dimension. By averaging the sensitivity information for these bound node families and thus reducing the dimensionality of the design space, this constraint significantly enhances the regularity of the primary stiffener layout and improves the structural integrity.

To validate the aforementioned customized design capability, the irregular nozzle example was further investigated. Optimization was conducted based on the initial grid without diagonal members shown in

Fig. 26, with an in-depth comparison of the evolutionary results with and without the collaborative nodal position constraint. All geometric and material parameters of the structure remained consistent with those in the previous sections. Fig. 27 compares the final physical layouts of the nozzle stiffened shell under these two customized requirements, while the corresponding optimization iteration convergence histories are presented in Fig. 28. The results clearly demonstrate that the proposed method can adaptively produce different stiffener configurations and achieve a significant and rational improvement in the structural fundamental frequency, even at the expense of sacrificing some geometric design domain degrees of freedom to satisfy specific engineering geometric constraints. This further supports the applicability of the proposed framework for customized anti-vibration design in engineering applications.

5.4. Orbital cabin-type curved shells

To further demonstrate the effectiveness of the multi-patch stitching technique introduced in Section 2 for optimization problems involving complex topologies and multiply connected stiffened shells, this section focuses on numerically maximizing the fundamental frequency of a simplified orbital cabin segment model based on a representative composite shell configuration. A typical spacecraft structure generally consists of an orbital module, a reentry module, and a propulsion module arranged in a tandem configuration. During the launch phase, the spacecraft serves as a payload for the launch vehicle, positioned at the foremost section within the payload fairing. Among these components, the orbital module typically sustains complex dynamic loads. Consequently, performing lightweight-oriented and fundamental-frequency optimization for such space-based composite curved shells can provide

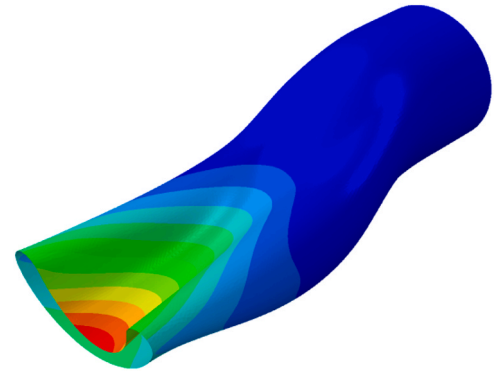


Fig. 22. First-order mode shape and fundamental frequency of the unstiffened irregular nozzle-like shell ($\Omega_1 = 785.83$ Hz).

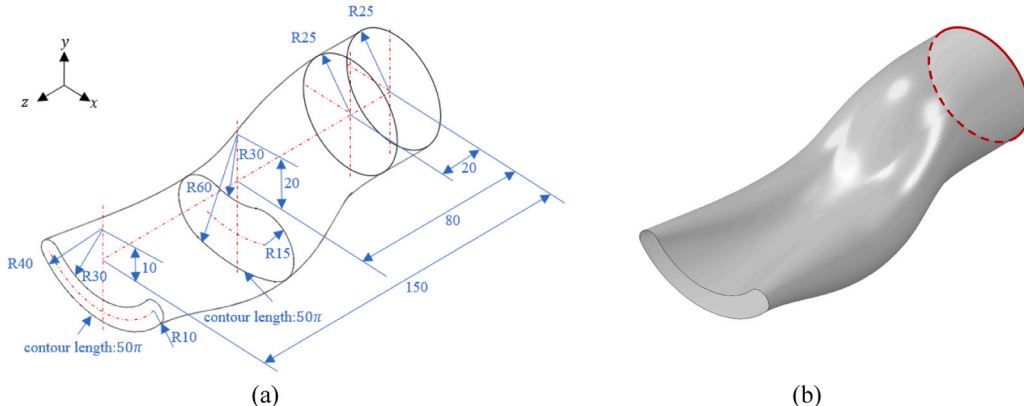


Fig. 21. Geometric model and boundary conditions of the irregular nozzle-like shell: (a) Geometric model; (b) Boundary conditions.

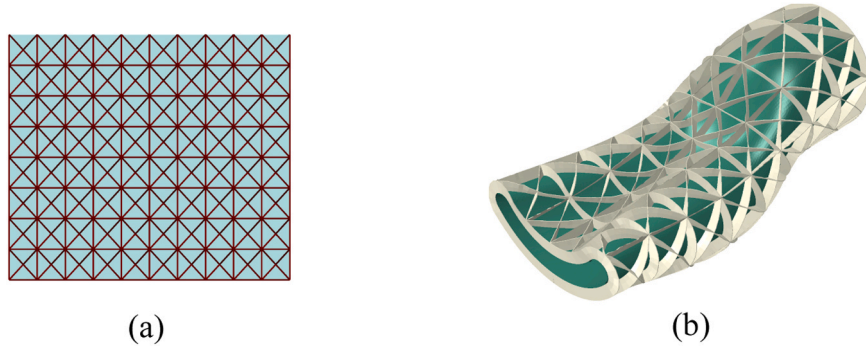


Fig. 23. Initial design of the stiffened irregular nozzle-like shell: (a) Parameter space; (b) Physical space.

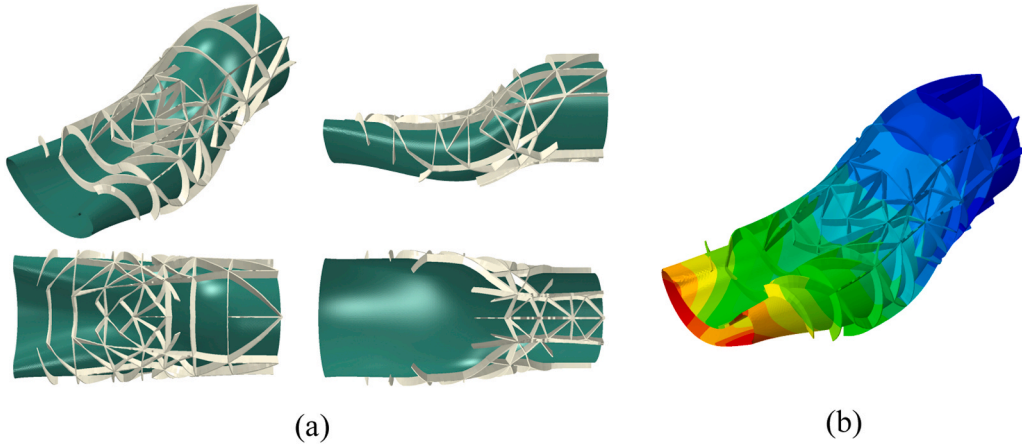


Fig. 24. Extracted layout of the stiffened irregular nozzle-like shell: (a) Geometric model; (b) First-order mode shape ($\Omega_1 = 1432.70$ Hz).

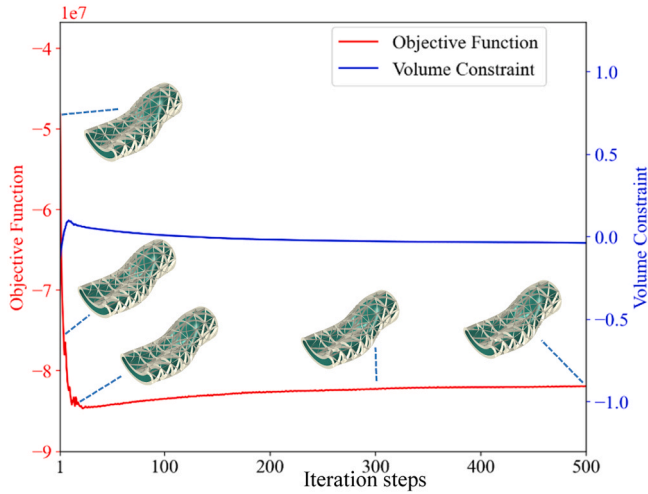


Fig. 25. Optimization iteration curve (continuous layout) and intermediate results of the stiffened nozzle shell, with $\Omega_1^1 = 1104.4$, $\Omega_1^5 = 1405.6$, $\Omega_1^{20} = 1462.3$, $\Omega_1^{300} = 1443.1$, $\Omega_1^{500} = 1440.4$.

a useful reference for subsequent engineering evaluations.

Based on the macroscopic geometry of a typical orbital cabin, this study abstracts and constructs the simplified geometric configuration and boundary conditions illustrated in Fig. 29. The model is assembled from a truncated conical shell, a cylindrical shell, and a spherical cap shell. Four fixed support nodes are uniformly distributed at the bottom ends on both sides of the cabin segment (indicated by the red areas in the figure). The thickness of the base skin for all curved shell segments is

uniform at $t_p = 4$ mm. The material properties for both the base skin and the stiffeners are defined as follows: elastic modulus $E_p = E_s = 73$ GPa, Poisson's ratio $\nu_p = \nu_s = 0.33$, and density $\rho_p = \rho_s = 2780$ kg/m³. The height of the stiffeners across all segments is fixed at $h_s = 10$ mm. Their continuous thickness serves as the design variable, with an initial value of $t_0 = 1$ mm and an optimization range set to $t \in [0.01, 4]$ mm and a stiffener removal threshold of $t_r = 0.05$ mm. Fig. 30 illustrates the initial first-order mode shape and the fundamental frequency of the orbital cabin-type curved shell (58.41 Hz). The volume fraction constraint imposed in this example is particularly stringent, requiring that the total volume of the stiffeners does not exceed 10% of the base skin volume (i. e., $\bar{V} = 0.1$).

Considering the symmetry of the structural geometry and boundary conditions, only one-quarter of the cabin structure was modeled for the actual optimization process to enhance computational efficiency. Based on the multi-patch stitching mapping mechanism described in Section 2.1, the spherical cap, cylindrical, and truncated conical segments of the orbital cabin were mapped onto three independent rectangular parametric domains. Since the driving nodes are strictly constrained to slide within their respective parametric domains, the optimization evolution of the stiffeners occurs exclusively within each individual segment. However, for the transition regions spanning across segment connections, geometric coupling of the stiffeners is achieved through shared nodes arranged on the common boundaries.

Fig. 31 illustrates the initial stiffener layout for this example, a configuration that reflects the traditional orthogonal empirical design commonly employed in aerospace stiffened shells. The entire quarter-structure model comprises 596 explicit stiffener components and 978 design variables. It is particularly noteworthy that for stiffeners whose skeletons coincide exactly with the common boundaries between cabin

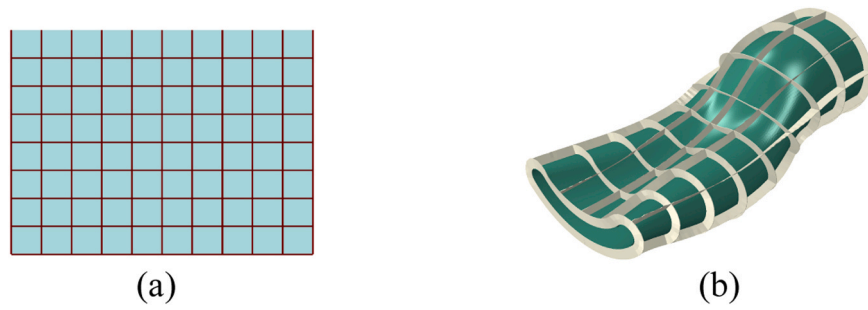


Fig. 26. Initial stiffener grid design without diagonal members: (a) Parameter space; (b) Physical space.

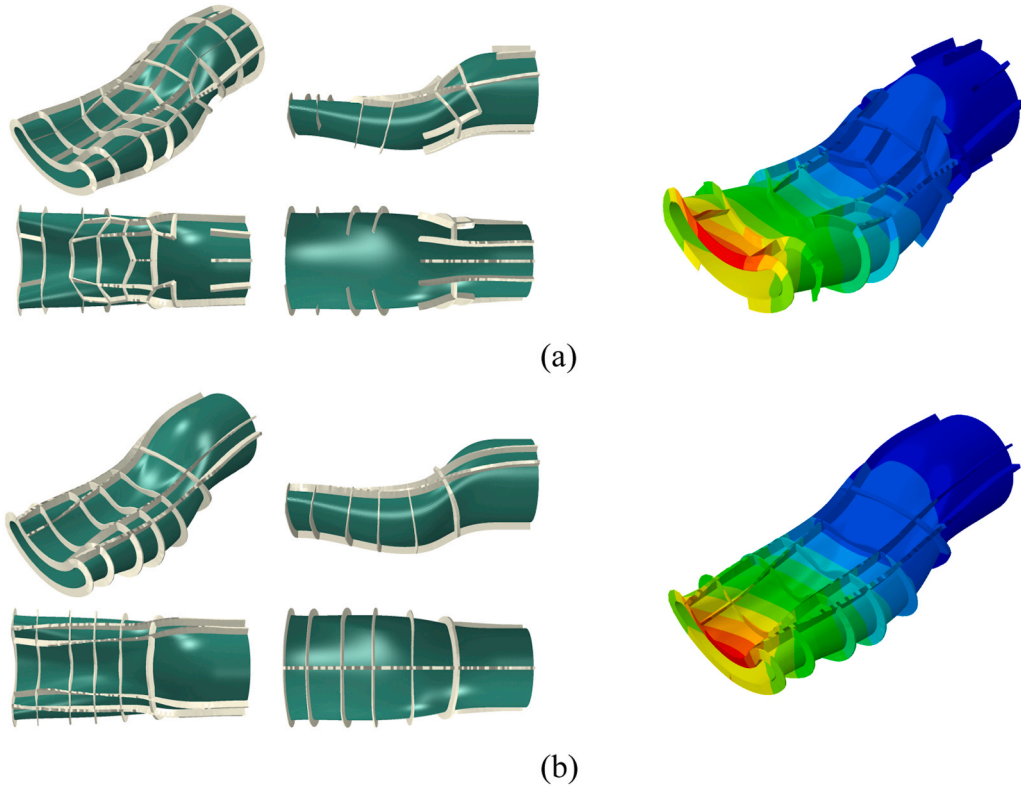


Fig. 27. Extracted layouts of the irregular nozzle-shaped stiffened shell under two different design requirements: (a) Final layout without collaborative nodal position constraints: geometric model (left) and first-order mode shape with $\Omega_1 = 1388.60$ Hz (right); (b) Final layout with collaborative nodal position constraints: geometric model (left) and first-order mode shape with $\Omega_1 = 1184.6$ Hz (right).

segments, the normal vector defining their height extension is uniformly assigned as the outward normal direction of the adjacent cylindrical surface. This convention ensures the uniqueness of the normal geometric construction.

Fig. 32 presents the continuous thickness optimization results for the stiffened orbital cabin-type curved shell, along with the extracted final layout after removing weakened stiffeners. The figure displays the full cabin structure, reconstructed by applying a symmetry mapping to the quarter-model optimization results. For visualization clarity, the thickness features of the stiffeners have been magnified by a factor of two. Regarding the material distribution pattern, the primary stiffeners with larger thicknesses are precisely concentrated in regions characterized by large amplitudes in the first-order fundamental mode and highly concentrated strain energy.

In the extracted layout, the phenomenon of disconnected stiffener segments is also observed. By comparing the fundamental frequency values of the overall structure before and after removing the weak stiffeners, it was found that the relative error between them is less than

0.01%. This extremely high degree of data agreement once again confirms the reasonableness of the extracted discrete layout, indicating that the weak stiffeners in low strain energy regions have been adaptively weakened by the algorithm, and their removal has a negligible impact on the overall fundamental frequency of the structure.

The iteration curves of the optimization process are shown in Fig. 33. It can be observed that in the early stage of optimization, the thicknesses of stiffeners in critical regions—which play a key role in enhancing the fundamental frequency of the stiffened shell—rapidly evolved towards the upper design limits, driving the curve to quickly reach an inflection point. Due to the structural symmetry and the small spatial deviation between the initial orthogonal grid and the regions of high modal strain energy, the driving nodes (those playing a significant role) required only a short search distance for optimization. This facilitated the optimization process to achieve rapid convergence.

This phenomenon indicates that the proposed method maintains good convergence behavior for shell surfaces involving complex multi-patch stitching. As a case study of a combined structure with

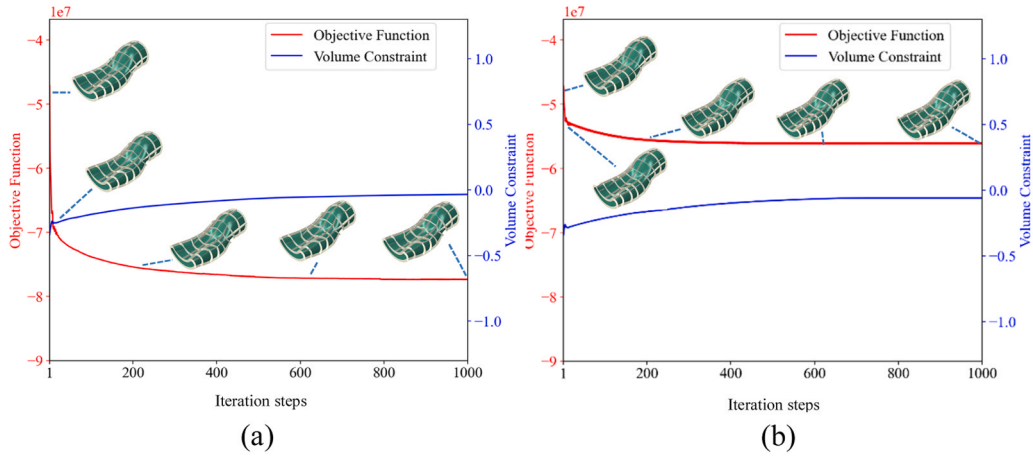


Fig. 28. Optimization iteration curves (continuous layout) and intermediate results of the irregular nozzle-shaped stiffened shell under two different requirements: (a) Iteration curve without collaborative nodal position constraints, with $\Omega_1^1 = 1092.7$, $\Omega_1^{20} = 1335.7$, $\Omega_1^{200} = 1382.0$, $\Omega_1^{600} = 1397.9$, $\Omega_1^{1000} = 1399.9$; (b) Iteration curve with collaborative nodal position constraints, with $\Omega_1^1 = 1092.7$, $\Omega_1^{20} = 1161.0$, $\Omega_1^{200} = 1187.3$, $\Omega_1^{600} = 1192.8$, $\Omega_1^{1000} = 1192.5$.

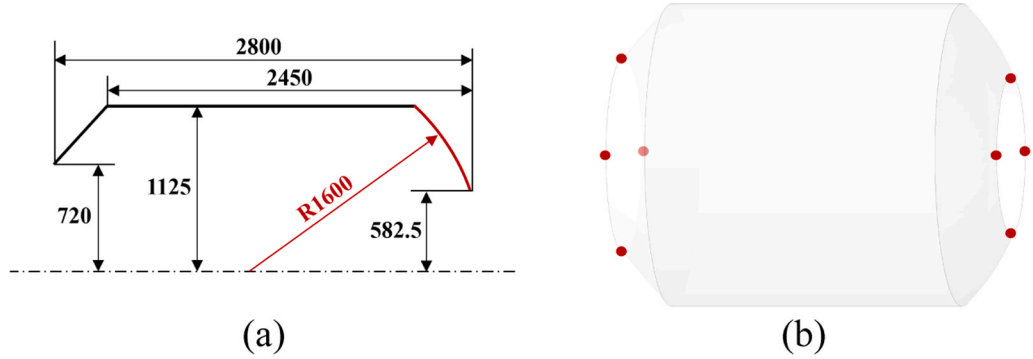


Fig. 29. Geometric model and boundary conditions of the orbital cabin-type curved shell: (a) Geometric model; (b) Boundary conditions.

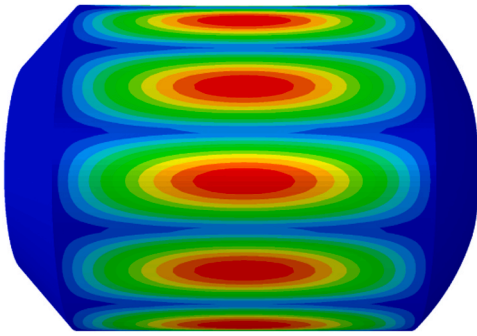


Fig. 30. Initial first-order mode shape and fundamental frequency ($\Omega_1 = 58.41\text{Hz}$) of the orbital cabin-type curved shell.

engineering relevance, these results support the feasibility and application potential of the proposed framework during the engineering conceptual design phase.

6. Conclusions and extensions

Based on the Moving Morphable Components (MMC) framework, this paper proposes an explicit topology optimization method for the design of stiffened thin-walled structures with complex surfaces aimed at maximizing their fundamental frequencies. To address the challenge of component mapping on three-dimensional complex surfaces, this study employs Computational Conformal Mapping (CCM) and multi-patch stitching techniques to construct a high-precision parametric model of the curved shell structure, laying a solid foundation for structural fundamental frequency optimization. Building upon this, this paper establishes an optimization model based on an adaptive initial

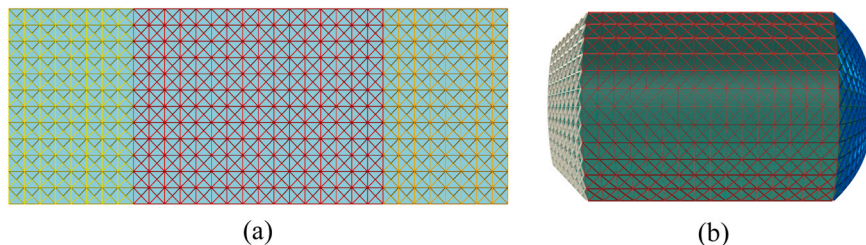


Fig. 31. Initial design of the stiffened orbital cabin-type curved shell: (a) Parametric space; (b) Physical space.

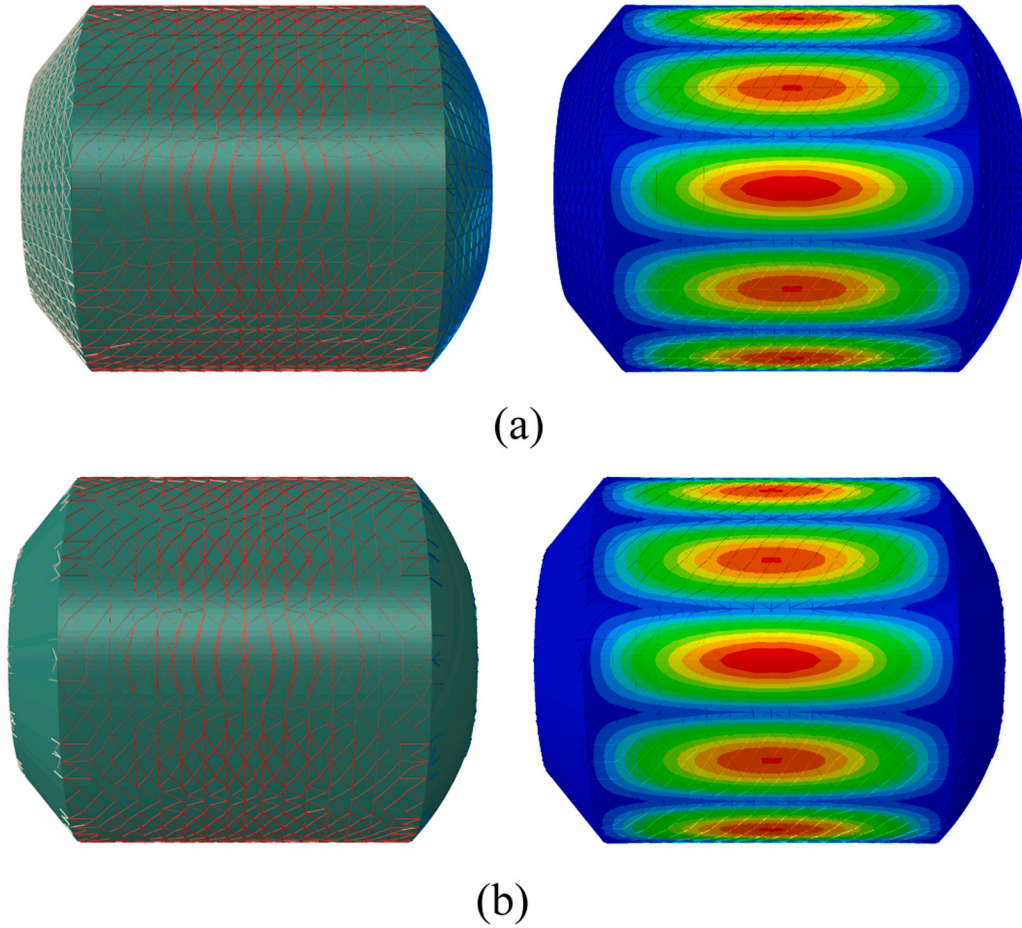


Fig. 32. Continuous layout and extracted layout of the stiffened orbital cabin-type curved shell: (a) Continuous layout ($\Omega_1 = 66.77$ Hz); (b) Extracted layout ($\Omega_1 = 66.78$ Hz).

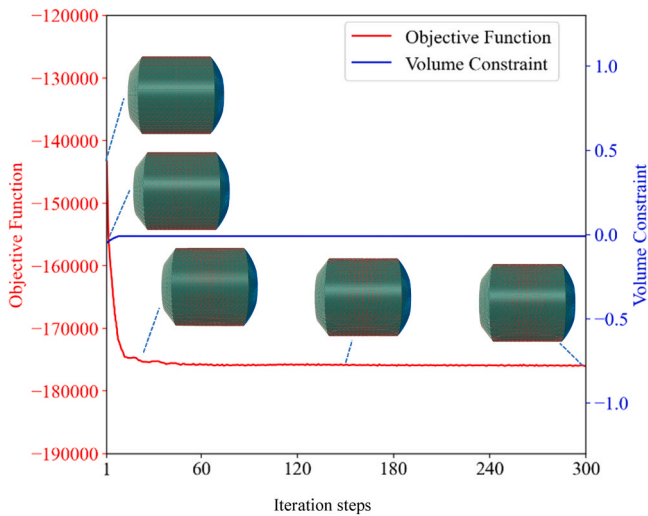


Fig. 33. Optimization iteration curves (continuous layout) and intermediate results of the stiffened orbital cabin-type curved shell, with $\Omega_1^1 = 60.248$, $\Omega_1^5 = 64.642$, $\Omega_1^{15} = 66.529$, $\Omega_1^{150} = 66.730$, $\Omega_1^{300} = 66.769$.

stiffener grid and a thickness-density dual penalty scheme. This scheme helps identify regions with relatively low modal strain energy and adaptively weaken or eliminate redundant stiffeners that contribute little to improving the fundamental frequency, thereby enabling the collaborative optimization of the spatial distribution and thickness

distribution of the stiffeners under prescribed height. A series of numerical examples involving complex curved shells demonstrate that the optimized stiffener configurations exhibit clear topological features and smooth boundaries, while achieving substantial improvements in the structural fundamental frequency under strict material constraints.

Compared to conventional implicit topology optimization methods, the proposed framework shows several theoretical and engineering advantages in the fundamental frequency optimization design of complex structures: First, by employing only the explicit geometric parameters of the stiffeners as design variables, it substantially reduces the degrees of freedom and solution scale of the eigenvalue optimization system. Second, the optimization results possess naturally clear topologies and well-defined boundaries, avoiding complex post-processing procedures commonly required by density-field results, such as grayscale thresholding and boundary reconstruction, and can facilitate subsequent CAD/CAE reconstruction and engineering assessment. To address the limitation of the current algorithm, wherein the final topological configuration depends to some extent on the initial grid layout, future work will introduce an adaptive component growth mechanism. This will allow stiffeners to dynamically generate at optimal locations guided by the dynamic sensitivity field during the optimization process, thus further broadening the design space and reducing dependence on the initial design.

Furthermore, the established explicit eigenvalue optimization framework possesses strong physical scalability. Given that both the determination of structural fundamental frequency and buckling critical loads fundamentally belong to generalized eigenvalue problems, this method can be extended in principle to the topology optimization for

buckling of complex stiffened thin-walled shells, enabling the synergistic design of structural stiffness and stability. The current optimization method does not yet consider variations in stiffener height; therefore, future work will further incorporate stiffener-height design variables to enhance the geometric flexibility and structural performance of the optimized stiffener layouts. Future research could further develop a multi-objective optimization framework that simultaneously considers fundamental frequency, buckling loads, and static stiffness, so as to better evaluate structural performance across multiple criteria for complex thin-walled stiffened shells in engineering applications. In addition, machine-learning-assisted surrogate or prediction models may be explored using accumulated optimization data to improve the efficiency of stiffener layout design for complex shells.

CRedit authorship contribution statement

Xu Guo: Writing – review & editing, Supervision, Resources, Project administration, Funding acquisition. **Wendong Huo:** Writing – review & editing, Validation, Methodology. **Jiaao Gao:** Writing – original draft, Visualization, Software, Investigation, Formal analysis, Data curation. **Chang Liu:** Writing – review & editing, Writing – original draft, Supervision, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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